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THE

PHYSICAL REVIEW.

ON MR. R. C. TOLMAN'S "PRINCIPLE OF SIMILITUDE."

By T. Ehrenfest-Afanassjewa.

§ 1. The principle which Mr. R. C. Tolman has developed in his two essays[1] on the above-mentioned subject already begins to attract attention.[2] Therefore it seems well to submit it to a more detailed examination. An accurate analysis shows that Tolman's considerations possess at least a close connection with the reduction to a definite hypothesis of the conviction of the homogeneity of all the equations of physics, a conviction which is commonly used without any foundation. This is not the intention of the author, as appears from his third paper on the same subject,[3] yet he really does nothing else but construct a system of dimensions of his own (indeed one that in some respects deviates from the C.G.S. system), and he examines all equations with a view to homogeneity *as regards this system of dimensions.*

I have already treated[4] from a considerably different point of view the mathematical relations which form the basis of all such considerations. The relation between the transformation with the aid of a model (miniature) system and the transformation of units is also treated there, nevertheless in the following lines I should like to treat this subject once more, and especially from the point of view from which Mr. Tolman considers it.

§ 2. *Introduction.*—We may imagine our universe to be transformed in such a way that every example of one and the same quantity Y_i is changed in one and the same ratio ξ_i (factor of transformation); the factors ξ_i, ξ_k, belonging to quantities of different kinds (for example

[1] R. C. Tolman, "The Principle of Similtude," Phys. Rev., *3*, 244 (1914); "The Specific Heat of Solids and the Principle of Similitude," Ibid., *4*, 145 (1914).

[2] G. Nordström, "R. C. Tolman's Prinzip der Aehnlichkeit und die Gravitation," *Oefversigt af Finska Vetenskaps—Societens Förhandlingar*, Bd. 58 (1914–15), afd. A, No. 22.

[3] R. C. Tolman, "The Principle of Similitude and the Principle of Dimensional Homogeneity," Phys. Rev., *6*, 219 (1915).

[4] T. Ehrenfest-Afanassjewa, "Der Dimensionsbegriff und der analytische Bau der physikalischen Gleichungen," *Math. Ann.*, *77*, 259, 1916.

say that Y_i defines masses and Y_h lengths) may in general be different. Then if our universe and this transformed universe are measured with *the same units*, for every number of our universe (x_i) there will be a corresponding number (x_i') in the transformed universe, the two numbers standing in the relation:

$$x_i' = \xi_i x_i.$$

If care has been taken that when the transformed numbers are inserted in the equations of physics these equations remain valid, so that all the numbers of one and the same equation are only changed in one and the same ratio (*i. e.*, are changed homogeneously[1]), then there will be a series of relations between $\xi_1, \xi_2, \cdots, \xi_i, \cdots$, which limits our choice of these quantities. It will depend on the structure of the equations and on their number, how many degrees of freedom (*i. e.*, how many independent ξ's) will be possessed by the transformation which we have defined, and also whether there are not more relations than can be fulfilled.

The equations which limit our choice of values for the ξ's are the following:

$$\xi_1^{\alpha_{e1}}\xi_2^{\alpha_{e2}} \cdots \xi_n^{\alpha_{en}} = \xi_1^{\beta_{e1}}\xi_2^{\beta_{e2}} \cdots \xi_n^{\beta_{en}} = \cdots, \tag{1}$$

$$\begin{aligned} \xi_1^{\rho_{e1}}\xi_2^{\rho_{e2}} \cdots \xi_n^{\rho_{en}} &= 1, \\ \xi_1^{\sigma_{e1}}\xi_2^{\sigma_{e2}} \cdots \xi_n^{\sigma_{en}} &= 1, \end{aligned} \tag{2}$$

where n is the total number of different quantities to be transformed which occur in the eth equation, $x_1^{\alpha_{e1}}, x_2^{\alpha_{e2}}, \cdots, x_n^{\alpha_{en}}$; $x_1^{\beta_{e1}}, x_2^{\beta_{e2}}, \cdots, x_n^{\beta_{en}}$ are the products of powers of these variables which occur *as factors* in the different members of the equation, and $x_1^{\rho_{e1}}, x_2^{\rho_{e2}}, \cdots, x_n^{\rho_{en}}$; $x_1^{\sigma_{e1}}, x_2^{\sigma_{e2}}, \cdots, x_n^{\sigma_{en}}$; $\cdots$ are products of powers which occur in any other way.[2] Besides which the additional conditions

$$\xi_n = \frac{\xi_v}{\xi_w^{\,k}} \cdots, \text{ etc.}, \tag{3}$$

if

$$x_n = \frac{d^k x_v}{dx_w} \cdots, \text{ etc.},$$

must not be neglected.

[1] For a more definite conception of "homogeneity" cf. my article, l. c., § 3.

[2] For example in the equation

$$y - a \sin (kx - mt) = 0$$

say $x_1 = y$, $x_2 = k$, $x_3 = x$, $x_4 = m$, $x_5 = t$, then if a is constant x_1 is the only "product of powers" which occurs *as a factor*; the second member has a constant as factor, consequently we obtain $\xi_1 = 1$ as the only equation of type (1). Further we obtain $\xi_2\xi_3 = 1$; $\xi_4\xi_5 = 1$ as the equations of type (2).

If the total number of equations (1), (2), (3) be of such a nature that there is at least one ξ not equal to 1 (and therefore arbitrary), then it will be possible to imagine a model (miniature) universe corresponding to our actual universe. In order to determine if this be the case, evidently it is not necessary to write down all the possible equations that occur in physics, but only that group (we may call them "fundamental equations") which is sufficient for the deduction of all other equations, for the latter will remain invariant if any transformation that the former allow is applied (provided any constants of integration which may occur be not also transformed, a consideration which greatly diminishes the scope of this theorem[1]).

After obtaining from the "fundamental" equations of physics the equations of the forms (1), (2) and (3) we may then obtain the general form that any derived equation (which does not contain a constant of integration) ought to possess. If the equation under consideration contains the variables $x_1, x_2, \cdots, x_n$ and if the relations between the corresponding ξ's can be reduced to the following form:

$$\begin{aligned}
\xi_{k+1} &= \xi_1^{a_{11}}\, \xi_2^{a_{12}} \cdots \xi_k^{a_{1k}},\\
\xi_{k+2} &= \xi_1^{a_{21}}\, \xi_2^{a_{22}} \cdots \xi_k^{a_{2k}},\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
\xi_{k+r} &= \xi_1^{a_{r1}}\, \xi_2^{a_{r2}} \cdots \xi_k^{a_{rk}},
\end{aligned}$$

then the desired equation can always be brought into the form

$$\phi\left(\frac{x_{k+1}}{x_1^{a_{11}} x_2^{a_{12}} \cdots x_k^{a_{1k}}},\ \frac{x_{k+2}}{x_1^{a_{21}} x_2^{a_{22}} \cdots x_k^{a_{2k}}},\ \cdots \frac{x_{k+r}}{x_1^{a_{r1}} x_2^{a_{r2}} \cdots x_k^{a_{rk}}}\right) = 0. \quad (5)$$

§ 3. The transition from the numbers x_i to x_i' may also be thought of in another way: instead of imagining measurements to be made with the same units in two different worlds, we may conceive the measurements to be carried out applying two different sets of units to the same objects ("in the same world"). If the units for each different kind of measurement be transformed independently, we shall obtain, using the original and the transformed set of units, the numbers

$$\begin{aligned}
x_1'' &= \xi_1'\, x_1,\\
x_2'' &= \xi_2'\, x_2.
\end{aligned}$$

These numbers will not in general satisfy the same equations (unless the coefficients before the products of powers which have been mentioned

[1] Cf. my article, l. c., § 5, Satz VII., Bemerkung I.

in equations (1) and (2) are corrected in corresponding ratios). If it is required that certain equations remain invariant, then between the factors ξ_i' relations must exist of the type of equations (1), (2) and (3). If now the units should be transformed in exactly the same way as the corresponding quantities in the model universe, the multipliers ξ_i' would be identically the same as the multipliers ξ_i (§ 2), and we should be sure that a system of dimensions would be determined such that, as regards the transformations allowed by it, all the equations of physics (which do not contain any constants independent of the "fundamental" equations (cf. § 2)) would be "physically homogeneous."

§ 4. Thus the widely spread conviction of the homogeneity of all physical equations with regard to the C.G.S. system would be reduced to the hypothesis of the possibility of a model universe, if one only could be sure:

(1) that a model universe in the sense defined above is possible,

(2) that we possess all equations which are wanted for a full description of the whole universe,

(3) that the latter condition is especially fulfilled by those equations which in the C.G.S. system serve to fix the dimensions of the different quantities.

Concerning this, the following may be remarked:

1. The transformation multipliers ξ_t, ξ_l, ξ_m, for the quantities time, length and mass are all fixed as independent without investigating, in accordance with the method described above, whether the model transformation actually does possess these three and only these three degrees of freedom.

2. The relations between the transformation multipliers (the "dimensions") of the other quantities are fixed in such a way that the equation in which a new quantity occurred for the first time (historically) remained invariant against the corresponding transformation (so that all coefficients were without dimensions). If in such a case in one equation two new quantities occurred in a common product, then the dimensions of one of them was fixed arbitrarily—so that the number of degrees of freedom of the transformation was diminished in an unnecessary way.[1]

3. No investigation was made of the question, whether there were any further equations between the same quantities, which should give a *necessary* reduction of the degrees of freedom. Of course as long as we meet only with equations such that each one introduces at least one new kind of quantity, it is not miraculous that all goes well. But is there any reason for believing that such equations will be sufficient to describe

[1] Cf. in Coulomb's equation, specific inductive capacity and electric charge.

the whole of nature? The gravitation-equation of Newton does not confirm it!

§ 5. It is always possible, however, to save the conviction of the existence of a model universe and therefore of the physical homogeneity of all the equations of physics (not only in the C.G.S. system, but in any other arbitrarily fixed system of dimensions), if one of the following artifices be used:

1. To the coefficient k_i of a product of powers

$$P_i = x_1^{a_{i_1}} x_2^{a_{i_2}} x_3^{a_{i_3}} \cdots x_n^{a_{i_n}}$$

which occurs in the equation under consideration and which destroys the required homogeneity, the meaning may be given of a physical quantity, the dimensions of which are fixed exactly by the condition that the whole member fulfill the requirement of homogeneity. In a model universe the number will then be correspondingly transformed. But it can also often be observed in our universe that such a number does not always remain the same in all circumstances (for example in the case of a constant of matter) and hence it may rightly be called a "quantity." A case of the destruction of homogeneity must occur in the C.G.S. system without any exception when a force is written as a function of space coördinates:

$$m\frac{d^2x}{dt^2} = f(x),$$

since according to our suppositions the right-hand side of the equation does not contain the time, and on the other side in the C.G.S. system the transformation multipliers of time and length are assumed to be independent.[1]

2. If it does not seem proper to regard the coefficient under consideration as a constant of matter, one can always have recourse to the following method: namely, to denote it as a product of special values of the variables occurring in the equation. There is an unlimited possibility of doing this.[2] But it will be obvious that *when the idea of the "physical*

[1] We may consider from this point of view the equation of elasticity, Coulomb's equation, and also the equation of gravitation. In this latter the coefficient under consideration has been resolved into two factors; *mass* times gravitational constant, the transformation multiplier, ξ_m for mass having already been defined as independent. If we compare with this Coulomb's electrostatic equation, we see that in both cases the function of the spatial coordinates is the same, but the factor of this function is treated in the two cases in an entirely different way. This is the reason why in Coulomb's equation one is not surprised by a consideration of the dimensions of the constant, whereas one always is astonished by the gravitational constant—as it has got dimensions.

[2] Cf. my article, l. c., § 9.

meaning" of the constants is extended in such a way, the supposition of a model universe and also of the physical homogeneity of all the equations of physics, becomes a mere triviality and ceases to afford a criterion for distinguishing between equations which are "physically allowable" and arbitrary equations.

If, on the contrary, we desire to leave untransformed all universal constants also in the model universe, then the question whether a model universe is possible or not has a definite meaning. But then, in order to determine the model transformation (and the appropriate system of dimensions) it is, besides other things, also necessary to include the equation of gravitation among the fundamental equations; in other words, in the corresponding system of dimensions the factors ξ_t, ξ_l, ξ_m, will not all be independent of each other.[1]

CONSIDERATIONS CONCERNING TOLMAN'S PRINCIPLE.

Tolman's "Principle of Similitude" leads to the hypothesis that all the equations of physics remain invariant (*i. e.*, with *constant* coefficients) when the units of measurement are transformed in such a way that certain appropriately fixed relations exist between the factors of transformation ξ_i. Tolman further indicates what these relations are to be—*so that he fixes his own system of dimensions.*

[1] The following remarks may be added in connection with this paragraph: *from the fact alone*, that in another (transformed) universe on measuring with *transformed* measuring units the same numbers may be obtained as in the original universe with the original units, we *have not yet any right to* conclude that on measuring with *transformed* units in the original universe we should obtain numbers which satisfy the original equations.

It is a triviality to say that a transformed universe is possible in which by the aid of transformed units the same numbers are obtained as in the original universe using the original units. (Cf. R. C. Tolman, "The Principle of Similitude," l. c., pp. 244, 245.)

Let us consider, for example, a two-dimensional world on a spherical surface. As long as the radius is considered to be a constant of this world there cannot be any geometrically similar figures, and the equation expressing the surface of a triangle in terms of the lengths of the sides is not homogeneous. If we proceed to a second world on a spherical surface of another radius, measuring with correspondingly changed units of length and surface, then the numbers thus obtained will satisfy the original equation, being simply the same numbers as obtained in the original world. But if we measure the original world with the transformed units, then the numbers obtained will *not* satisfy the original equations unless we agree to measure the radius (which figures as a constant quantity) also with the transformed units; that is to say, a constant, *universally constant* for the world under consideration, *will now have to get another numerical value.*

An analogy exists, as regards calculation, between the above example and the following. Consider along with our universe another in which the gravitational field may have another intensity. Suppose the remaining quantities in the gravitational equation and the corresponding units to be transformed in *suitable* proportions: the universe thus obtained and measured will satisfy the original gravitational equation, the constant of gravitation being of course the same number as in the original case. If, however, in the original universe measurements are to be made with the new units, one becomes aware that the constant of gravitation has not the same value in both universes (that the field intensities are not the same).

If this system is compared with the C.G.S. system, the following things appear:

In the composition of Tolman's system for the most part the same equations are used as "fundamental" as in the C.G.S. system. Also he has put aside the equation of gravitation, so that also here all variables which occur in it have already got their dimensions in another way, and a real constant has to be used to remove the difference in dimensions of both members. But, and in this point the theory of Tolman differs from the C.G.S. system, two other assumptions are made which diminish the number of degrees of freedom from three to one—the assumption of the constancy of velocity and of the unit of electric charge.[1]

When we add to Tolman's list of fundamental equations the equation of gravitation, then the number of degrees of freedom for the model transformation is reduced to *zero*, in other words *the equations of physics are of such a nature that no model transformation exists in which all the universal constants have the same values as in our universe.*

Another method of looking at the problem is possible, which is also adopted by G. Nordström[2] in the article already mentioned. It consists in maintaining at all costs the possibility of a model universe and, in accordance with this, of "correcting" all physical equations to make them satisfy the principle of similitude. Mr. Nordström succeeds (at least as far as his researches have gone) in doing this by replacing Newton's equation of gravitation by another one, in which instead of "inertial mass" there occurs another quantity, "gravitational mass."

It seems impossible to say anything against such a point of view: experiment alone must decide which of such equations are to be admitted as the true ones.

Of course the question may be allowed: does the application of the "Principle of Similitude" succeed when it is used for determining the relation between the radius and mass of the planets? (in analogy to the relation between the mass and radius of an electron obtained by Mr. Tolman).[3]

[1] The former is necessary as a consequence of equations (2) § 2, if one accepts among the equations of physics the equations of the theory of relativity, in which the member $\sqrt{1-v^2/c^2}$ occurs, and if one assumes that the number c remains a universal constant. The second is not required by any fundamental equation.

[2] Cf. § 1, note 2.

[3] R. C. Tolman, l. c., 3, pp. 251, 252.

LEIDEN,
January 15, 1916.

NOTE ON THE HOMOGENEITY OF PHYSICAL EQUATIONS.

BY RICHARD C. TOLMAN.

THE manuscript of the preceding article was kindly sent to me by the author so that I could state my position in regard to it.

Mrs. Ehrenfest-Afanassjewa[1] has presented in this paper a very satisfactory and general treatment of the conditions which must be fulfilled if a set of physical equations are to be homogeneous with respect to a given transformation, that is, if they are to remain unchanged when each of the quantities of a particular kind Y_i occurring in the equations is multiplied by a factor ξ_i. Equations (1), (2) and (3) in her article show the method of determining how many of the factors ξ_1, ξ_2, ξ_3, etc., for the different kinds of quantities Y_1, Y_2, Y_3, etc., can be taken arbitrarily and still have the transformation a homogeneous one for all the equations in the set under consideration. She has also shown that the possibility of constructing a miniature (model) universe, in which all the laws of physical behavior would be identical even to numerical coefficients with those in our own universe, depends on the possibility of finding a homogeneous transformation for *all* the fundamental equations of physics in which at least one of the factors ξ_1 can be taken as arbitrary. And with these conclusions I agree entirely.

Mrs. Ehrenfest-Afanassjewa has further pointed out that the C.G.S. system of dimensions prescribes homogeneity for physical equations when a transformation is carried out in which three of the multipliers ξ_l, ξ_m, ξ_t are arbitrary and that my own principle of similitude requires homogeneity with respect to a transformation in which only one of the multipliers is taken arbitrarily. I agree of course entirely with such a form of statement for the principles of dimensional homogeneity and of similitude, as will be seen in my paper comparing these two principles.[2]

Because of this possibility of expressing in such a similar form the requirements of the two principles, Mrs. Ehrenfest-Afanassjewa is inclined to speak of the theory of similitude (cf. § 1) as being merely another system of dimensions differing from the C.G.S. system. Although this method of speaking is perhaps logically possible, it seems to me to be *very undesirable* since I do not believe that the principle of similitude determines what we ordinarily mean by a set of dimensions.

[1] T. Ehrenfest-Afanassjewa, this journal, (1916).

[2] Tolman, PHYS. REV., *6*, 219 (1916).

The dimensions of a quantity may be best regarded, I believe, as a shorthand statement of the definition of that kind of quantity in terms of certain fundamental kinds of quantity, and hence also as an expression of the essential physical nature of the quantity in question. If, for example, we define force as mass times acceleration, the dimensions of force will be $[mlt^{-2}]$ and this may be regarded as a shorthand recapitulation of the definition of force in terms of mass, length and time, and also as an expression of the essential physical nature of force.

The reason, now, why certain physical equations have to be dimensionally homogeneous is because in the cases under consideration the physical nature of the quantities equated has to be the same. If, for example, we know that the centripetal force acting on a particle depends on its mass m, tangential velocity v, and radius of rotation r, we know that the particular combination of m, v and r which determines the centripetal force will have to have the physical nature of a force and hence the dimensions of force. And since the only combination of these quantities which has the right dimensions is $(mv^2)/r$ we know that the equation for centripetal force must be

$$f = k\frac{mv^2}{r},$$

where the numerical constant turns out to have the value unity.

In contradiction to such considerations as the above, I do not believe that the theory of similitude can be regarded as furnishing a system of dimensions or shorthand statements of the physical nature of quantities. If we should try to regard the principle of similitude as determining a system of dimensions, we should be obliged to say, as will be seen from Table II. in my article already referred to (l. c., p. 226), that force, for example, has the dimensions $[l^{-2}]$ and since this is neither an adequate shorthand statement of the definition of force, nor a satisfactory description of the essential physical nature of force I believe we shall do well if we do not speak of the theory of similitude as determining a system of dimensions, but rather realize that there is a real and fundamental difference between the principle of similitude and the principle of dimensional homogeneity.

A few words with regard to this difference in the fundamental nature of the two principles will not be out of place. As I have already pointed out (l. c.) there are a large number of important physical equations which do *not* stand in "intrinsic" agreement with the principle of dimensional homogeneity.[1] Thus in Stefan's law connecting the energy density

[1] These are equations which cannot be derived with the help of the principle of dimensional homogeneity, but, nevertheless, as I have shown (l. c.) often can be treated with the help of the principle of similitude.

u in a hohlraum with the temperature T

$$u = kT^4$$

there is no identity between the physical nature of energy density and of temperature to the fourth power, and only a "formal" agreement with the principle of dimensional homogeneity can be brought about by arbitrarily assigning to the constant k the dimensions $[ml^{-1}t^{-2}T^{-4}]$. This somewhat artificial procedure of assigning dimensions to such a numerical constant may be useful in determining what changes in the numerical value of the constant will be brought about by a change in units of measurement, but it does not change the fact that in equations such as these we have an equality between the numbers on the two sides but no identity in the physical nature of the terms equated, and hence no "intrinsic" agreement with the principle of dimensional homogeneity.

Since there are these important physical equations which do not stand in intrinsic agreement with the principle of dimensional homogeneity, we do not obtain from this principle any information as to the possibility of constructing a miniature universe in which *all* the laws of physics would be the same even to *numerical constants* as those in our own universe. For as will be seen from the work of Mrs. Ehrenfest-Afanassjewa, the possibility of constructing such a universe depends on the possibility of finding a set of multipliers for physical quantities such that all the fundamental equations of physics will undergo a homogeneous transformation, *at least one of the multipliers being chosen arbitrarily.* And all that we have learned from the principle of dimensional homogeneity with regard to this matter, is that if we take all five of the multipliers (ξ_l, ξ_t, ξ_m, ξ_μ and ξ_T) arbitrarily, there are some important relations between physical quantities which will not be transformed homogeneously.

In contradistinction to dimensional considerations, which have given us no information as to the possibility of constructing a miniature universe, the theory of similitude definitely postulates this possibility, and hence the possibility of finding a homogeneous transformation for all the equations of physics, at least one of the multipliers ξ_i being chosen arbitrarily. Thus I agree with Mrs. Ehrenfest-Afanassjewa that *all* the "fundamental" equations of physics must agree with the theory of similitude if my fundamental postulate is really valid. Moreover, this agreement with the principle of similitude must be an "intrinsic" agreement in the sense in which we have already used that word, and this necessity for "intrinsic" agreement arises because in building the miniature universe we are to use by hypothesis the same materials as in our

actual universe[1] and yet the laws of physics are to be the same even to numerical coefficients in the miniature universe as in our actual universe. For this reason I agree with Mrs. Ehrenfest-Afanassjewa (cf. § 5), as will be seen from my very first paper,[2] that the fundamental equation for gravitational considerations must also agree with the principle of similitude. And since Newton's law of gravitation does not agree with the principle of similitude, we are forced, as I have already stated, either to the conclusion that the principle of similitude has like the principle of dimensional homogeneity only a restricted validity and a limited field of usefulness, or to the conclusion that Newton's law of gravitation is not a satisfactory statement of the fundamental equation for gravitational considerations. I am inclined of course to this latter alternative, and have pointed out two possibilities for the reconciliation of Newton's law with the theory of similitude, namely, either (1) that the gravitational constant is not a true numerical constant but is dependent on the properties of some unknown gravitational mechanism, or (2) that the force of attraction between the gravitating bodies is not in reality necessarily proportional to ordinary mass, but to some other quantity ("gravitational mass") whose transformation relations would differ from those of ordinary mass. If this were the case it would be merely an accidental coincidence that in the phenomena with which we are already acquainted "gravitational mass" happens to be proportional to "inertial mass." It is this latter of these possible solutions which has been adopted by Nordström in his article[3] "R. C. Tolman's Prinzip der Aehnlichkeit und die Gravitation." Such considerations as these make it very desirable to determine if electrons or rays of light have a weight proportional to their mass, and hence I am inclined to agree with Mrs. Ehrenfest-Afanassjewa's feeling (cf. § 5) that the universal validity of the principle of similitude is to a certain extent a matter for experimental investigation.[4]

[1] It is this necessity of using the same materials for building the miniature universe which forces us to use in this construction a free space in which the velocity of light is unchanged and to use electrons which have the same charge as in the actual universe. This determines the transformation relations for velocity and for electric charge. (Cf. Mrs. Ehrenfest-Afanassjewa, footnote 12.)

[2] Tolman, PHYS. REV., *3*, 244 (1914).

[3] Nordström, *Öfversigt af Finska Vetenkaps-Societens Förhandlingar*, 57, Afd-A, No. 22 (1915).

[4] As to Mrs. Ehrenfest-Afanssjewa's closing question, I presume that it must have been asked in a spirit of pleasantry. Of course the reason why the principle of similitude cannot be used to derive a physical law showing the relation between the mass and radius of a planet is because there is no physical law connecting the mass and radius of a planet. We could construct planets of any mass and radius that we desired. The actual planets which happen to be in our solar system have masses and radii which were determined by past astronomical accidents, whose history is still a sealed book even to believers in the principle of similitude.

BERKELEY, CAL., U. S. A.

March 7, 1916.

RESISTANCE AND THERMO-ELECTRIC RELATIONS IN IRON OXIDE.

BY C. C. BIDWELL.

ALTHOUGH the electrical properties of "variable conductors," *i. e.*, the weak electro-positive elements such as carbon, silicon, titanium, tellurium, etc., and the oxides and sulphides of the noble metals, have been rather extensively studied by Königsberger[1] and those working with him, there is still room for considerable work in this field. The present paper offers some results on artificially prepared specimens of Fe_3O_4 and shows in particular a possible cross-relation between thermo-electric power and electrical resistance.

Most of the work of Königsberger on the oxides of iron deals with naturally occurring crystals of large size, the behavior along the different axes being studied. Königsberger offers an explanation of the behavior of this class of substances on the basis of the electron theory of Drude. The number of free electrons within the substance is assumed to increase with the temperature according to an exponential law, whereas, in the case of the metals proper, it is assumed to be constant. By making use of certain empirical relations he arrives at the expression

$$\rho = A(1 + \alpha t + \beta t^2)e^{\frac{Q}{RT}}. \qquad (1)$$

(ρ) is the specific resistance; (t), the temperature on the Centrigrade scale; (T), the temperature on the absolute scale; (R), the gas constants, approximately (2); Q, the heat of dissociation of the electrons, *i. e.*, the heat required to liberate a gram-atom of electrons from the atoms to which they are supposed to be bound; (e), the Naperian base; (A), (α) and (β), constants.

In the present work the exponential (e^{aT}) was found to be as satisfactory, empirically, as the series ($1 + \alpha t + \beta t^2$) and more convenient for graphical testing. Königsberger expression thus becomes

$$\rho = Ae^{\left(\frac{Q}{RT} + aT\right)}, \qquad (2)$$

where (a) is a constant. For the sake of graphical testing we may write this

[1] Königsberger, Ann. d. Phys., 32, p. 179, 1910.

$$\text{Log}_{10}\,\rho = .435\left(\frac{Q}{RT} + aT\right) + \log_{10} A, \tag{3}$$

and, by differentiation,

$$\left(-\frac{1}{\rho}\frac{d\rho}{dT}\right) = \frac{Q}{RT^2} - a. \tag{4}$$

If a straight line is obtained by plotting the left-hand member of (4) as ordinates against $1/T^2$ as abscissa, the equation is verified. From such a line, if obtained, the constants (Q), (R) and (a) can be found. A further check may then be obtained by plotting equation (3), using $\log_{10}\rho$ as ordinates and $[(Q/RT) + aT]$ as abscissa. These lines afford a good test of equation (2).

PREPARATION OF SPECIMENS AND METHODS EMPLOYED.

Specimens of Fe_3O_4 were prepared by fusing c.p. Fe_2O_3 by means of a carbon arc. The fused mass on cooling was ground in a porcelain mortar, yielding a gray, magnetic powder. Metallic iron, if present, would have shown itself as small spherical droplets, incapable of being ground to the almost impalpable powder finally obtained. By means of a hydraulic press, blocks were made of this material with thermo-junction leads of platinum and platinum-rhodium compressed in. These blocks were baked at a red heat and then cut to the dimensions of rods about one centimeter long by two millimeters thick, with the junctions firmly embedded in the ends. They were then baked in the open air for an hour or more at approximately 1400° C.

For resistance measurements the specimen was placed in a platinum-wound tubular furnace, current sent through the specimen by means of leads (b, b') (Fig. 1) and the potential fall measured across the leads (a, a'). The potential fall was also measured across a known resistance in series with the leads (b, b'). The direction of current was then reversed and the measurements repeated. The temperatures at A and B usually differed by not more than 10° at 1400° C. and 3° at 800° C. The thermal E.M.F. was corrected for by the reversal of current.

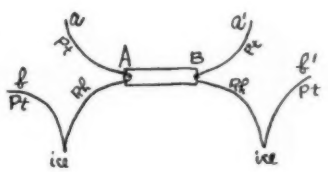

Fig. 1.

For the measurements of thermal E.M.F. the specimen was hung in a short vertically wound furnace in which a large gradient could be obtained. The gradient along the specimen was made uniform by encasing it in a quartz tube, the end of which was open and flush with the lower end of

the specimen. A gradient of from 30° to 80° was used in the thermo-electric measurements. At the same mean temperature, trials were made with several different temperature gradients along the specimen. The same thermo-electric power was found for a given average temperature, no matter what the gradient was. The gradient was therefore made as large as possible that inaccuracies in measurements might be proportionately small.

Owing to the effect of high temperature on the physical condition or state of the specimen, the order in which measurements were taken was of great importance. Measurements were taken, say for resistance, going from high to low temperature and then for thermal E.M.F. going from low back to high temperatures. In this way the specimen was not subjected to extreme high temperatures between the resistance run and the thermo-electric run. The resistance and thermo-electric data therefore referred to the same physical state and could be compared. It was found that this was not the case if the extreme high temperature came between the two runs.

Discussion of Data.

Figure 2 shows resistance plotted directly against temperature. The portion to the extreme right is plotted to the scale marked on the y-axis,

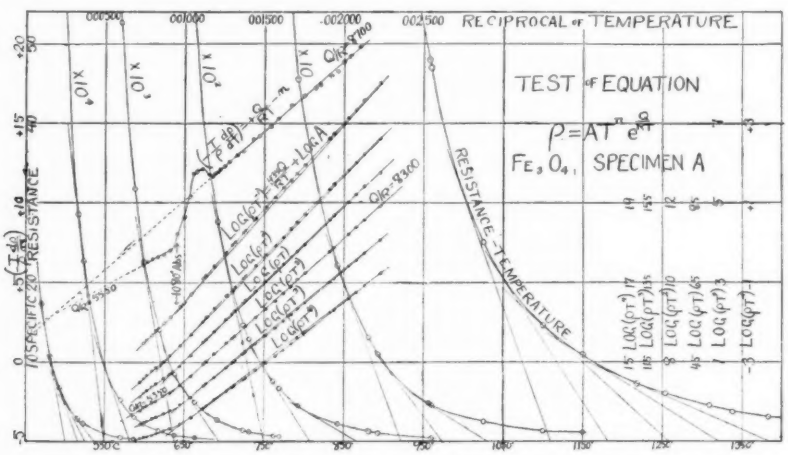

Fig. 2.

for each succeeding portion to the left the scale on the y-axis is multiplied by 10. Tangents are drawn at intervals and from the data so obtained equation (4) is plotted (see Fig. 3). A straight line is found up to about 600° C. A reversible transformation takes place between 600° C. and

800° C. The slope is different beyond 800° C. The exact transformation temperature depends somewhat upon the previous heat treatment of the specimen. Six different specimens were prepared and studied and this behavior found in every case. The transformation may be related to the recalescence phenomenon in iron which occurs in this region. It

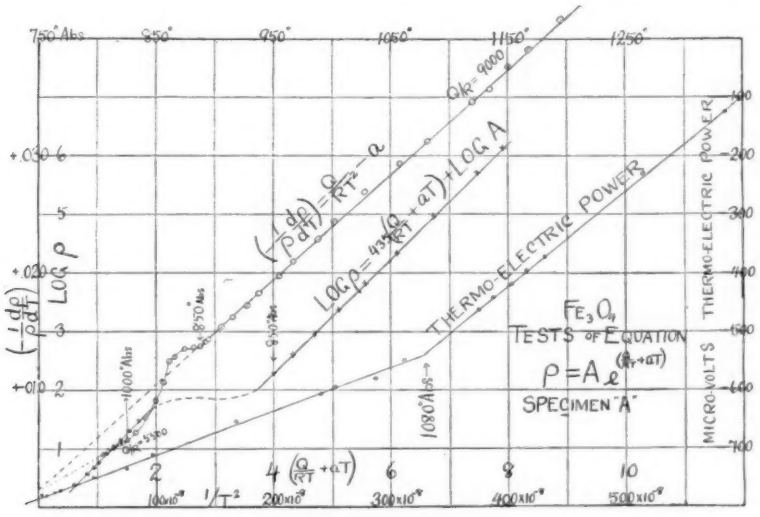

Fig. 3.

cannot however be ascribed to the accidental presence of metallic iron, since in some cases the specimen was heated in open air for several hours at 1400° C. Traces of iron would certainly have been oxidized.

From the curve for equation (4) the values of the constants (Q/R) and (a) were obtained, and equation (3) plotted (Fig. 3). Again we have straight lines showing the transformation as before. Fig. 4 shows the behavior of a second sample.

If the specimen is held in the neighborhood of 1400° C. for an hour or more a new temperature-resistance curve will be obtained. This however will again give straight lines for equations (3) and (4) but with slightly different constants.

Since the equation of Königsberger is so largely empirical we might legitimately ask if there are other expressions which will satisfy the data equally well. For reasons which will appear the following equation was tried out.

$$\rho = AT^n e^{\frac{Q}{RT}}. \tag{5}$$

This may readily be tested in the form

$$\log_{10}(\rho T^{-n}) = \frac{.435Q}{RT} + \log_{10} A. \tag{6}$$

Values of (n) from ($-$ 4) to ($+$ 1) were assumed and the corresponding curves plotted (Fig. 2). The data are well satisfied with $n = -2$. Values of n below ($-$ 2) give a line with a curvature in one direction, above ($-$ 2) with a curvature in the other direction.

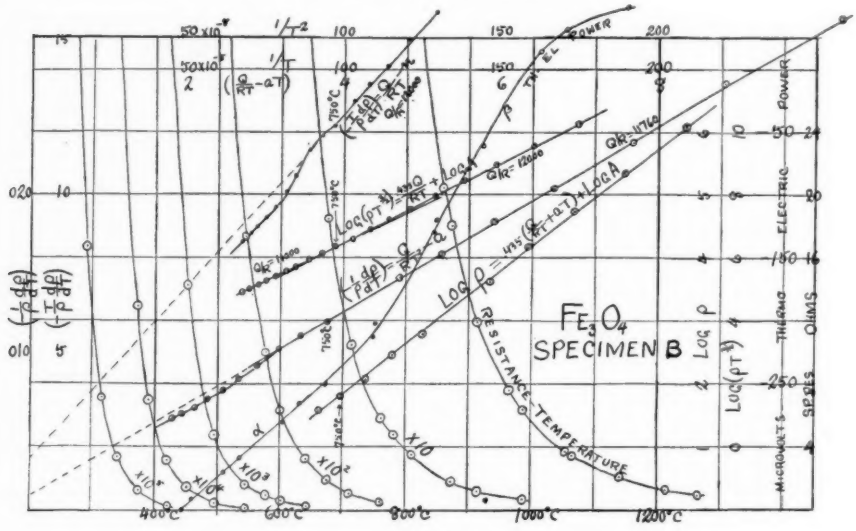

Fig. 4.

By differentiation we get

$$\left(-\frac{T}{\rho}\frac{d\rho}{dT}\right) = \frac{Q}{RT} - n. \tag{7}$$

Plotting $\left(-\frac{T}{\rho}\frac{d\rho}{dT}\right)$ against ($1/T$) we get a straight line in which the intercept on the y-axis is about ($+$ 1.5) which gives a value of (n) about ($-$ 1.5). The values of $d\rho/dT$ are only approximate, depending upon what one considers the best tangents, and hence the method can only approximately fix the value of (n). The log curves would be equally well satisfied with $n = -1.5$. The transformation points are brought out by these curves also.

If the data agree with both equation (4) and equation (7), it follows that $-a = -n/T$, that is, that either (a) or (n) is a variable. If $n = -2$, the values of (a) lie between $-$ .0022 and $-$ .0050. In this case the line for equation (4) should be slightly concave toward the x-axis. In case (a) were the constant and (n) the variable, the line for equation (7) should be concave toward the x-axis. In either case the deviation is too slight to be shown by the present data. Equation (5) does not reduce to the expression for pure metallic conduction when we assume (N) constant and thereby $e^{Q/RT} = 1$, as do equations (1) and (2). Equation

(5) however is of great convenience since it can be plotted in the log. form directly from the original data. From this plot, the value of (Q) may be determined for a given specimen, and transformation points and other peculiarities brought out.

THERMO-ELECTRIC RELATIONS.

The relation between thermo-electric power and temperature is shown also on Figs. 3 and 4. The transformation point observed on the resistance lines is corroborated by the thermo-electric data. The data give two straight lines intersecting at or near the transformation temperature as given by the resistance lines. These lines are only approximately straight or at least only straight through the interval shown, since with several specimens deviations from the straight lines became apparent at the higher temperatures. The same thermo-electric line may be obtained over and over again with a given specimen if one does not go above the highest temperature of the initial run. If that temperature is exceeded however another set of straight lines is obtained.

A series of thermo-electric power lines and resistance lines taken on the same specimen is shown in Figs. 5, 6, 7. The resistance curves are plotted in accordance with equation (6) with $n = -1.5$. The specimen was first heated for two hours at 1200° C., then the measurements plotted

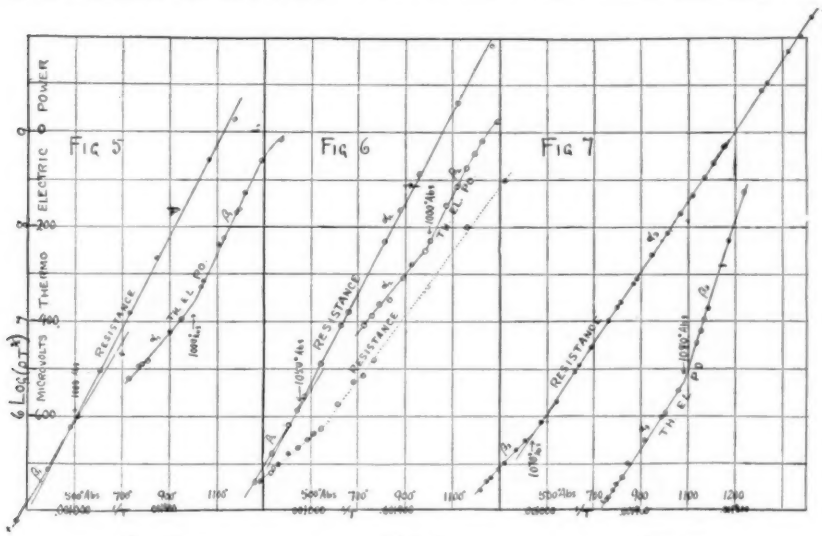

Fig. 5. Fig. 6. Fig. 7.

in Fig. 5 were taken. The specimen was then heated to 1400° for a few minutes and the resistance run shown dotted in Fig. 6 was taken. Attempts to get thermo-electric data revealed the fact that the specimen

was in a very unstable condition. The thermo-electric power was much larger in absolute value (lower on the curve) but was very erratic, not remaining constant even at constant temperature. The specimen was allowed to stand at room temperature until the next day. Conditions were then found to be perfectly stable and the resistance and thermo-electric data shown in Fig. 6 were obtained. It is seen that the specimen had reverted nearly to the condition shown in Fig. 5. The specimen was then again heated to 1400° C. and held there for two hours. Thereafter the conditions shown in Fig. 7 were found.

A study of these curves brings out rather interesting evidence concerning a relation between (Q), the electron heat of dissociation, so-called by Königsberger, and the rate of change of thermo-electric power. One would expect if (Q) were decreased the thermo-electric power would increase more rapidly. Assuming that the thermo-electric power is due to the difference of the pressure of the free electrons in the oxide and in the platinum and that the platinum has the larger number and hence the greater pressure, then, as the temperature goes up and the electrons are liberated in the oxide the pressure difference becomes less and the thermo-electric power decreases. With such a state of affairs an increase in (Q) will cause the thermo-electric power to increase less rapidly and vice versa. A rough check on this is apparently shown by this data (Figs. 5, 6, 7). The lower temperature modification is indicated by (α), the higher temperature form by (β). The slopes of the thermo-electric power lines for the (α) modifications and for the (β) modifications are plotted (Fig. 8) against the corresponding values of

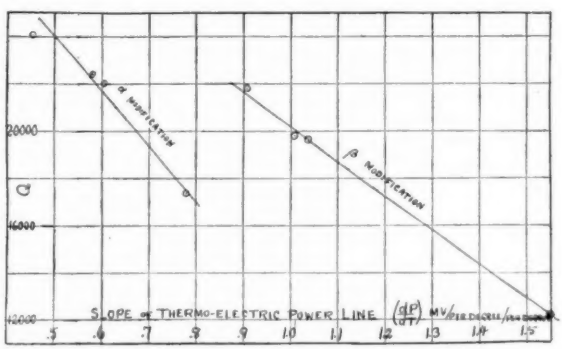

Fig. 8.

(Q) as obtained from the log-resistance curves. The upper point shown on each curve is taken from specimen (B), Fig. 4, the other points all refer to the runs shown in Figs. 5, 6, 7, specimen (A).

Additional work is needed to establish this relation but the evidence is thought to be suggestive. The change in slope of the thermo-electric line going from the (α) to the (β) modification does not correspond proportionately to the change in (Q), the transformation appearing to effect the two quantities differently. If, however, we consider the changes produced by heat treatment in either one of the modifications, there does appear to be a relation between the two quantities. Further work along this line is in progress.

This work was carried on under a grant from the Carnegie Institute of Washington.

CORNELL UNIVERSITY,
1916.

THE DISPERSION OF HYDROGEN AND HELIUM ON BOHR'S THEORY.

BY C. DAVISSON.

THE theory of atomic and molecular structure proposed by Dr. N. Bohr[1] has had such marked success in accounting for a variety of atomic and molecular phenomena, that it seems desirable to test it in as many ways as possible. With this end in view the following calculations were undertaken to discover if the molecules of hydrogen and helium proposed by Bohr are capable of accounting for the observed dispersion curves of the corresponding gases. It is assumed in the calculation that the phenomenon of dispersion is correctly accounted for on the ordinary electronic theory of Drude and Lorentz.

HYDROGEN.

The hydrogen molecule on Bohr's theory is a system of two positively charged nuclei, P_1 and P_2 (Fig. 1) each of charge e, and two electrons, the latter rotating in a circular orbit about the axis P_1P_2. The geometry of the system in its state of steady motion is that required for equilibrium upon ordinary electrodynamic principles, while definite dimensions are secured to the molecule by the hypothesis that the angular momentum of each electron is $h/2\pi$, where h is Planck's radiation constant.

Denoting by a, one half the distance between the nuclei; by r, the radius of the orbit, and by ω, the angular velocity of each electron, the above mentioned conditions require

$$a = \frac{1}{9 - \sqrt{3}} \cdot \frac{h^2}{\pi^2 me^2} = 2.98 \times 10^{-9} \text{ cm.},$$

$$r = \sqrt{3}a = 5.17 \times 10^{-9} \text{ cm.},$$

$$\omega = (14 - 3\sqrt{3}) \frac{\pi^3 me^4}{h^3} = 4.39 \times 10^{16} \text{ sec}^{-1},$$

where the following values have been used for the constants

$$h = 6.62 \times 10^{-27} \text{ gm. cm.}^2/\text{sec.},$$

$$e = 4.77 \times 10^{-10} \text{ E.S. units},$$

$$m = 8.99 \times 10^{-28} \text{ gm.}$$

[1] Phil. Mag., XXVI., p. 1 (1913); XXVI., p. 476 (1913); XXVI., p. 857 (1913); XXX., p. 394 (1915).

In discussing the effect of a train of light waves upon such a system it will be helpful to imagine a set of rectangular coordinates having its origin at the center of symmetry of the molecule, its z axis parallel to the electric vector of the waves, and its x axis so chosen that the plane, $y = 0$ includes the axis P_1P_2.

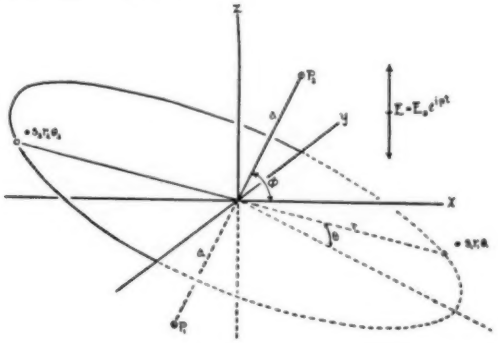

Fig. 1.

Let the intensity of the electric vector at the molecule considered be represented by $E = E_0\epsilon^{ipt}$. This is practically the total electric intensity as the part arising from the polarization of the medium is negligible in comparison. The principal effect of this field upon the system will be to disturb the motion of the electrons, so that at a given instant their positions will be $s_1r_1\theta_1$ and $s_2r_2\theta_2$, where s_1 and s_2 are their displacements from the equilibrium plane of the orbit in the direction P_1P_2, r_1 and r_2 their new radial distances, and θ_1 and θ_2 their angular positions about P_1P_2 referred to the plane $y = 0$.

Equations of motion for the electrons may be conveniently obtained by writing down expressions for T and W, the kinetic and potential energies of the system and applying Hamilton's Principle. Assuming the nuclei at rest

$$T = \tfrac{1}{2}m(\dot{s}_1^2 + \dot{s}_2^2 + \dot{r}_1^2 + \dot{r}_2^2 + r_1^2\dot{\theta}_1^2 + r_2^2\dot{\theta}_2^2),$$

$$W = e^2\left(\frac{1}{d} - \frac{1}{d_{11}} - \frac{1}{d_{12}} - \frac{1}{d_{21}} - \frac{1}{d_{22}}\right) + eE(z_1 + z_2),$$

where d is the distance between the electrons, d_{12} is the distance between the electron $s_1r_1\theta_1$ and the nucleus P_2, etc.

Performing the operation

$$\delta\int_{t_1}^{t_2}(T - W)dt = 0,$$

the following equations are obtained:

$$(1)\quad m\ddot{s}_1 + e^2\left\{\frac{s_2 - s_1}{d^3} - \frac{a - s_1}{d_{12}{}^3} + \frac{a + s_1}{d_{11}{}^3}\right\} + eE\sin\varphi = 0,$$

$$(2)\quad m\ddot{s}_2 + e^2\left\{\frac{s_1 - s_2}{d^3} - \frac{a - s_2}{d_{22}{}^3} + \frac{a + s_2}{d_{21}{}^3}\right\} + eE\sin\varphi = 0,$$

$$(3)\quad m\frac{d}{dt}(r_1{}^2\dot{\theta}_1) + \frac{e^2 r_1 r_2\sin(\theta_2 - \theta_1)}{d^3} + eEr_1\sin\theta_1\cos\varphi = 0,$$

$$(4)\quad m\frac{d}{dt}(r_2{}^2\dot{\theta}_2) + \frac{e^2 r_1 r_2\sin(\theta_1 - \theta_2)}{d^3} + eEr_2\sin\theta_2\cos\varphi = 0,$$

$$(5)\quad m(\ddot{r}_1 - r_1\dot{\theta}_1{}^2) + e^2\left\{\frac{r_2\cos(\theta_2 - \theta_1) - r_1}{d^3} + \frac{r_1}{d_{12}{}^3} + \frac{r_1}{d_{11}{}^3}\right\} - eE\cos\theta_1\cos\varphi = 0,$$

$$(6)\quad m(\ddot{r}_2 - r_2\dot{\theta}_2{}^2) + e^2\left\{\frac{r_1\cos(\theta_1 - \theta_2) - r_2}{d^3} + \frac{r_2}{d_{22}{}^3} + \frac{r_2}{d_{31}{}^3}\right\} - eE\cos\theta_2\cos\varphi = 0,$$

where φ is the angle between P_1P_2 and the plane $z = 0$.

The complete solution of these equations is unnecessary, as it is obvious from considerations of symmetry that the average displacement at right angles to the field taken over a large number of fortuitously oriented molecules must be negligible in comparison with that parallel to the field. If the displacements of the electrons have components $\xi_1\eta_1\zeta_1$ and $\xi_2\eta_2\zeta_2$ in the rectangular coördinates, we are concerned only with the quantity $(\zeta_1 + \zeta_2)$, or $(z_1 + z_2)$ since the later is zero for the undisturbed motion of the electrons. Denoting by z the quantity $(z_1 + z_2)$

$$z = (s_1 + s_2)\sin\varphi - (r_1\cos\theta_1 + r_2\cos\theta_2)\cos\varphi.$$

and $(-e)z$ measures the polarization of the molecule parallel to the field.

The s, r and θ displacements of the electrons will, in general, have components parallel to z. Indicating these by z_s, z_r and z_θ

$$z = z_s + z_r + z_\theta.$$

An approximate solution for z may then be obtained by regarding these contributions as mutually independent. That such an approximation is permissible is indicated by the independence of refractive indices and light intensities.

To obtain an expression for z_s assume

$$\begin{cases} r_1 = r_2 = r, \\ \theta_1 = \theta_2 - \pi = \omega t + \epsilon, \end{cases}$$

where ϵ is the initial phase of the first electron. Then

$$z = z_s = (s_1 + s_2)\sin\phi$$

and equations (1) and (2) when added become

$$m(\ddot{s}_1 + \ddot{s}_2) + e^2 \left\{ \frac{a + s_1}{(4a^2 + 2as_1)^{\frac{3}{2}}} + \frac{a + s_2}{(4a^2 + 2as_2)^{\frac{3}{2}}} - \frac{a - s_1}{(4a^2 - 2as_1)^{\frac{3}{2}}} - \frac{a - s_2}{(4a^2 - 2as_2)^{\frac{3}{2}}} \right\} + 2eE_0\epsilon^{ipt} \sin \varphi = 0.$$

Expanding the expression multiplying e^2 and neglecting terms in the second and higher powers of s_1/a and s_2/a this reduces to

$$m(\ddot{s}_1 + \ddot{s}_2) + \frac{e^2}{16a^3}(s_1 + s_2) + 2eE_0\epsilon^{ipt} \sin \varphi = 0,$$

for which the solution is

$$(s_1 + s_2) = -\frac{2\, e/m \sin \varphi}{p_s^2 - p^2} E_0\epsilon^{ipt} + \text{Complementary Solution},$$

where $p_s^2 = e^2/16ma^3$ is the square of 2π times the natural frequency of the system for s vibrations. As the complementary solution contains an arbitrary phase angle its effect over any considerable number of molecules may be expected to disappear regardless of damping and may be neglected. We have, therefore,

$$z_s = (s_1 + s_2) \sin \varphi = -\frac{2\, e/m \sin^2 \varphi}{p_s^2 - p^2} E_0\epsilon^{ipt}.$$

To obtain an expression for z_θ assume

$$s_1 = s_2 = 0,$$
$$r_1 = r_2 = r,$$
$$\theta_1 = \omega t + \epsilon + \alpha_1,$$
$$\theta_2 = \omega t + \pi + \epsilon + \alpha_2,$$

α_1 and α_2 being the angular displacements due to the field.

Then

$$z = z_\theta = r(\alpha_1 - \alpha_2) \sin (\omega t + \epsilon) \cos \varphi$$

and equations (3) and (4) when subtracted and approximated become

$$mr^2(\ddot{\alpha}_1 - \ddot{\alpha}_2) + \frac{e^2}{4r}(\alpha_1 - \alpha_2) + er \cos \varphi \{ \sin (\omega t + pt + \epsilon) + \sin (\omega t - pt + \epsilon) \} E_0\epsilon^{ipt} = 0.$$

The particular solution of this equation combined with the above expression for z_θ gives

$$z_\theta = -\frac{e}{m} \left\{ \frac{\sin (\omega t + pt + \epsilon)}{p_\theta^2 - (\omega + p)^2} + \frac{\sin (\omega t - pt + \epsilon)}{p_\theta^2 - (\omega - p)^2} \right\} \sin (\omega t + \epsilon) \cos^2 \phi E_0\epsilon^{ipt},$$

where $p_\theta^2 = e^2/4mr^3$ is the square of 2π times the natural frequency of the electrons for vibrations tangential to the orbit.

A similar treatment of the r contribution to z leads to an expression for z_r containing an imaginary natural frequency. This is due to the instability of the molecule for r displacements contributing to z_r. Bohr has pointed out that this instability disappears if it be assumed that the angular momentum of each electron remains constant for its disturbed, as well as for its steady motion.

If we make use of this assumption rather than ordinary electro-dynamic principles the r and θ motions will be connected by the relation

$$r_1^2\dot{\theta}_1 = r_2^2\dot{\theta}_2 = r^2\omega.$$

Substituting for r_1 and r_2 the values $(r + \rho_1)$ and $(r + \rho_2)$, and for $\dot{\theta}_1$ and $\dot{\theta}_2$, $(\omega + \dot{\alpha}_1)$ and $(\omega + \dot{\alpha}_2)$ we have for small displacements

$$(\rho_1 - \rho_2) = \frac{r}{2\omega}(\dot{\alpha}_2 - \dot{\alpha}_1).$$

The value for z_r is

$$z_r = (\rho_2 - \rho_1)\cos(\omega t + \epsilon)\cos\varphi$$

$$= \frac{r}{2\omega}(\dot{\alpha}_1 - \alpha_2)\cos(\omega t + \epsilon)\cos\varphi.$$

Combining this equation with the solution of the above differential equation differentiated once with respect to time we obtain

$$z_r = -\frac{e}{2m\omega}\left\{\frac{(\omega + p)\cos(\omega t + pt + \epsilon)}{p_\theta^2 - (\omega + p)^2} + \frac{(\omega - p)\cos(\omega t - pt + \epsilon)}{p_\theta^2 - (\omega - p)^2}\right\}$$
$$\cos(\omega t + \epsilon)\cos^2\varphi E_0\epsilon^{ipt}.$$

Adding the expressions for z_s, z_θ and z_r the sum is the value of z for a molecule whose axis makes an angle φ with the plane $z = 0$ and whose first electron has an initial phase ϵ. In a large collection of molecules the proportion having these characteristics $\pm \frac{1}{2}d\varphi$ in the first case, and $\pm \frac{1}{2}d\epsilon$ in the second will be

$$\frac{\mathrm{I}}{4\pi}\cos\varphi d\varphi d\epsilon$$

and the average value of z, say $\bar{z}$, will be given by

$$\bar{z} = \frac{\mathrm{I}}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{+\pi/2} z\cos\varphi d\varphi d\epsilon.$$

Performing the integration

$$\bar{z} = -\frac{2}{3}\frac{e}{m}\left\{\frac{1}{p_s^2 - p^2} + \frac{\frac{1}{2} + \frac{\omega + p}{4\omega}}{p_\theta^2 - (\omega + p)^2} + \frac{\frac{1}{2} + \frac{\omega - p}{4\omega}}{p_\theta^2 - (\omega - p)^2}\right\} E_0 \epsilon^{ipt}.$$

If there are N molecules per c.c. of the gas the polarization is given by

$$P = N(-e)\bar{z}$$

and the dielectric constant, k, for the frequency $p/2\pi$ is

$$k = 1 + 4\pi\frac{P}{E}.$$

If μ is the refractive index of the medium for this frequency

$$\mu = \sqrt{k}$$

and

$$(\mu - 1) = 2\pi\frac{P}{E},$$

since P/E is small compared with unity. Substituting

$$(\mu - 1) = \frac{1}{3\pi}\frac{Ne^2}{m}\left\{\frac{1}{\nu_s^2 - \nu^2} + \frac{\frac{1}{2} + \frac{W + \nu}{4W}}{\nu_\theta^2 - (W + \nu)^2} + \frac{\frac{1}{2} + \frac{W - \nu}{4W}}{\nu_\theta^2 - (W - \nu)^2}\right\},$$

where $2\pi W$ has been written in for ω, $2\pi\nu$ for p, etc.; W being the frequency of rotation of the electrons, ν the frequency of the light vibrations, etc.

It will be observed that $(\mu - 1)$ becomes infinite, indicating absorption bands, for

$$\nu = \nu_s$$

and for

$$\nu = W - \nu_\theta, \quad \text{or} \quad \nu = \nu_\theta - W.$$

Using for N the value 2.7×10^{19} and for the other constants the values already mentioned the constants of the dispersion equation are:

$$\frac{1}{3\pi}\frac{Ne^2}{m} = 7.26 \times 10^{26},$$

$$\nu_s^2 = 1.51 \times 10^{31},$$

$$\nu_\theta^2 = 1.16 \times 10^{31},$$

$$(W - \nu_\theta)^2 = 1.28 \times 10^{31}.$$

The empirical equation for $(\mu - 1)$ determined by C. and M. Cuthbertson,[1] has the form

[1] Proc. Roy. Soc., *A* lxxxiii, p. 166 (1909); *A* lxxxiv, p. 13 (1910).

$$(\mu - 1) = A\left(\frac{1}{\nu_0^2 - \nu^2}\right),$$

the values of the constants being

$$A = 16.92 \times 10^{26},$$

$$\nu_0^2 = 1.2409 \times 10^{31}.$$

The fair agreement between the values for ν_0^2 and $(W - \nu_\theta)^2$ probably has no significance as the dispersion term in which the latter occurs is of an essentially different type from that found experimentally; while the very great discrepancy between the coefficients of the frequency terms in the two cases is distinctly disappointing. In fact, it seems impossible on any reasonable supposition, such as supposing one or more of the degrees of freedom inhibited, to obtain even an approximate representation of the experimentally determined relation.

There are perhaps one or two points of interest in the calculated results apart from their relation to the empirical equation. If Drude's rule[1] for determining the number of dispersing electrons per molecule corresponding to a given natural frequency be applied to the coefficient it will be found to give 2/3 instead of two. This is due to the fact that for each natural frequency the electrons have but one degree of freedom each, instead of three, as is usually assumed in the general dispersion theory. It would seem that what Drude's calculation really gives is $(nd/3)$, where n is the number of electrons and d is the number of degrees of freedom per electron for the natural frequency under consideration. Another feature of the calculated expression is a type of dispersion term in which the absorption band corresponds to no single natural frequency of the system but to the difference of two frequencies.

HELIUM.

The theoretical dispersion equation for helium subject to the same assumptions may be got very readily from the results already obtained for hydrogen. The helium molecule on Bohr's theory is identical with that of hydrogen, so far as these calculations are concerned, except that

$$a = 0,$$

$$r = 3.09 \times 10^{-9} \text{ cm.},$$

$$\omega = 12.2 \times 10^{16} \text{ sec.}^{-1}.$$

The form of the dispersion equation will therefore be the same. The coefficient of the frequency terms is also unaltered. The values of

[1] Ann. der Phys. (1904).

the calculated constants and those determined experimentally by C. and M. Cuthbertson[1] follow:

Calculated	*Observed*
$\frac{1}{3\pi}\frac{Ne^2}{m} = 7.26 \times 10^{26}$	$A = 12.12 \times 10^{26}$
$\nu_s^2 = 7.03 \times 10^{31}$	$\nu_\theta^2 = 3.499 \times 10^{31}$
$\nu_\theta^2 = 5.41 \times 10^{31}$	
$(W - \nu_\theta)^2 = 14.6 \times 10^{31}$	

Again the calculated values seem quite incapable of accounting for those observed.

It is not impossible, of course, that this lack of agreement is due to a mistaken conception of the mechanism of dispersion.

CARNEGIE INST. OF TECH.,
PITTSBURGH, PA.

[1] Loc. cit.

ON THE INITIAL CONDITION OF THE CORONA DISCHARGE.

BY JAKOB KUNZ.

THE glow discharge of electricity surrounding the transmission wires under the influence of high potential differences has been studied by electrical engineers of England and America in recent years. In the majority of these investigations alternating current has been used, and very definite empirical results have been obtained. Comparatively few researches have been carried out on the direct corona, among which those of Watson, Schaffers and S. P. Farwell[1] may be mentioned. Farwell especially has shown that the phenomena are far more complicated than what has been revealed by the use of alternating potentials.

Before making an attempt at an explanation of some of the phenomena I wish to describe briefly the various forms of the corona due to direct current potentials. There is hardly another electric phenomenon which shows the difference between positive and negative electricity in so many different ways as the corona. There are electric, optical and mechanical differences. For very small wires the negative glow appears before the positive, for larger sizes the positive glow appears before the negative. The boundary between the two regions is a diameter of about 0.075 mm. The positive corona in air forms a very even uniform layer of light of practically constant thickness. The negative corona on the other hand starts also in a uniform layer of red light, but very quickly breaks up into bright beads, separated from each other by dark intervals. Especially at lower pressures this difference between positive and negative polarity is very conspicuous. The negative beads distribute themselves in nearly equal intervals and are fairly stable, so that they can be photographed readily. The positive discharge from the wire in a coaxial cylinder has never been found to break up into beads, but if the discharge takes place between two parallel wires, then at higher potentials the positive column of light also breaks up into shorter intervals and finally into beads. The question arises as to whether the beads are connected with irregularities on the surface of the wire, or whether it is an intrinsic phenomenon, independent of the surface irregularities. The number of beads depends on

[1] Watson, Electrician, London, Vol. 63, p. 828, 1909; Vol. 64, p. 707 and 776, 1909–10.
Schaffers, Comptes rendus, July, 1913, p. 203.
S. P. Farwell, Transactions of American Institute of Electrical Engineers, Nov. 13, 1914.

the pressure of the gas and on the potential difference. If these two variables are kept constant, the number of beads remains constant, indicating that the beads on smooth wire are an intrinsic phenomenon of the negative discharge. In addition special experiments have been performed with polished and chemically corroded silver wires in order to test this conclusion. For a given potential difference the corona current increases with decreasing diameter of the wire, and with decreasing pressure.

The characteristic curve, like the starting point of the corona, depends on the polarity and diameter of the wire. For wire smaller than 0.077 mm. diameter the current from the negative wire is greater than that from the positive wire. For the diameter of 0.077 mm. the currents for opposite polarity coincide accurately over a certain range of voltages above the critical voltage, and then the negative current becomes and remains the larger. For sizes of wire larger than 0.077 mm. the curves for the two signs cross each other. For the lower potential differences the positive current is the greater, and for higher potential differences the negative current is the greater. Previous investigators already found that the relation between the electric force E at the surface of the wire and the radius R_1 is given by the formula

$$E = E_0 + \frac{b}{\sqrt{R_1}},$$

where E_0 and b are constants, E is calculated by the electrostatic formula:

$$E = \frac{\Delta V}{R_1 \log \frac{R_2}{R_1}}$$

ΔV being the potential difference between the central wire and the cylinder of radius R_2. For the smallest sizes of wires used, the relation between E and R_1 ceases to hold as can be seen from Table I.

The critical voltage required to produce visible corona depends not only on the radius of the wire, but also on the pressure. At very low pressures the negative corona starts before the positive one, at higher pressures the positive corona starts at first. The characteristic curves also depend on the pressure. The pressure of the gas and the radius of the wire play an analogous rôle, very thin wires seem to correspond to low pressures. The relation between the pressure p of air, the radius R_1 of the central wire and the critical electric force E at the surface of the wire is given as follows:

$$E = p\left(E_0 + \frac{b}{\sqrt{pR_1}}\right),$$

where E_0 and b are constants. This relation holds as far down as 53 mm. Hg pressure for the positive corona: the constants E_0 and b have different values for the positive and negative wire.

TABLE I.

R cm.	V+Volts.	E+Volts per cm.	E+Calcul.	V—Volts.	E—Volts per cm.	E—Calcul.
0.00135	2,720	2.74×10^5	2.62	2,520	2.52×10^5	2.55
0.00218	3,380	2.58	2.29	3,230	2.45	2.23
0.0023	3,500	2.25	2.09	3,300	2.08	2.04
0.00258	3,630	2.12	1.99	3,500	2.02	1.94
0.00386	4,060	1.66	1.67	4,060	1.66	1.65
0.00678	5,140	1.31	1.34	5,320	1.36	1.33
0.00825	5,710	1.25	1.25	6,140	1.21	1.21
0.012	6,600	1.07	1.09	6,840	1.09	1.09
0.013	7,180	1.07	1.06	7,660	1.14	1.06
0.0205	8,900	0.93	0.91	9,370	0.99	0.92
0.0325	10,880	0.80	0.79	11,440	0.83	0.80
0.0385	11,850	0.77	0.75	12,400	0.79	0.76
0.0512	13,500	0.71	0.69	14,120	0.73	0.71
0.0642	14,700	0.65	0.65	15,220	0.64	0.64

When the corona starts, the pressure of the gas increases suddenly. We shall call this pressure ionization pressure. It can easily be measured by means of a sensitive U-tube open manometer. This increase of the pressure is very distinctly different from the increase of the pressure due to the evolution of Joule's heat. As soon as the current is interrupted the ionization pressure sinks suddenly down to zero, while the other pressure increases and dies out gradually. The ionization pressure is in general for a given potential difference larger when the wire is negative than when it is positive, but the difference is very small, if not opposite at the beginning of the corona. The ionization pressure is very nearly proportional to the current, especially when the wire is positive.

The ionization pressure as well as the fact that a higher potential difference is necessary to start the corona for thicker wires can be used with advantage for the construction of voltmeters, some of which are in use in the laboratory of the University of Illinois.

It has been mentioned that the negative electricity leaves the wire in the form of very beautiful beads or brushes, mostly evenly spaced along the wire. The number of brushes per unit length depends on the pressure and on the potential difference. With increasing pressure and with increasing potential difference the number of beads per unit length increases and their brightness at the same time decreases. The beads start from a point of the wire and spread out fanlike in a plane at right

angles to the wire. Very interesting is the influence of a short arc in series with the tube upon the character of the positive and negative discharges. The very well defined positive layer of light spreads out considerably and the negative brushes disappear almost entirely, giving room to a continuous glow, whose boundary is ill defined; in other words, a very short spark in series with the discharge tube destroys the difference in the appearance between the positive and the negative corona. This is due to the superposition of a high frequency alternating or intermittent current. A small change of the spark length between the corona and the dynamos produces very marked differences in the luminous discharge. For a certain spark length the corona assumed the form of bright streamers which fill the entire space between the wire and the cylinder. If the spark length is slightly changed, these streamers concentrate into a few luminous bands, about equally spaced, whirling round the wire and presenting a very beautiful aspect. If the potential difference is slightly increased, this phenomenon is replaced by the arc, which is apparently the more stable form of discharge. With the introduction of a spark a hissing sound will be heard from the corona tube.

The difference between positive and negative elctricity makes itself felt finally in mechanical effects. When the corona takes place between two parallel wires which are not stretched too strongly, the negative wire bows in toward the positive and the positive bows away from the negative. When the wires are purposely made rather slack the positive wire vibrates strongly with a circular motion, while the negative wire remains motionless.

The field between two parallel wires and between a cylinder and a coaxial wire has been explored by means of a third platinum electrode. Even before the corona started there was found a distortion of the electrostatic field especially in the neighborhood of the electrodes; and in many if not in all cases the electric force at the surface of the wires is different from the calculated value in the moment when the corona arises. The observed electric force seems to be larger than that calculated from the electrostatic formula. When the field is studied in the space between the central wire and the coaxial cylinder, it becomes very difficult to explore the neighborhood of the negative wire, where the potential seems to be subject to continuous changes. The exploration of the field around the positive wire offers no difficulties.

The explanation of the large variety of phenomena described is far from being complete. An attempt at an explanation of some of the phenomena will be made. One might expect that luminous discharge begins when the electric force or polarization on the surface of the wire

obtains a constant value required for the ionization of the molecules. It has been found however that the critical electric force is given by the expression:

$$E_1 = E_0 + \frac{b}{\sqrt{R_1}}$$

Various values for E_0 and b have been given, for instance, $E_0 = 30$, $b = 9$ by Y. S. Townsend. Farwell found that the values of E_0 and b are distinctly different for positive and negative wires. For positive wires he found $E_0 = 31.6$; $b = 8.43$. For negative wires $E_0 = 35.0$; $b = 8.06$. We shall now assume that in the neighborhood of the wire in a layer of constant thickness δ a certain constant energy is required for the beginning corona, different for positive and negative electricity; indeed the splitting up of the molecules into ions and the emission of light requires energy. When a sufficient amount of energy is supplied, the luminous discharge called corona will occur. It has been shown by Schaffers that the thickness $\delta = 0.07$ cm. of the luminous layer is independent of the radius of the wire. In the neighborhood of the wire, the electric force E assumes large values so that the polarization also is large and an opposing electric force E_0 will be created, so that the resultant electric force is equal to $E - E_0$. If k is the dielectric constant, R_1 the radius of the wire, then we have:

$$E_g = \frac{k}{8\pi} 2\pi R_1 \delta (E - E_0)^2,$$

$$E = E_0 \sqrt{\frac{4E_0}{kR_1\delta}}.$$

If E_g, k and δ remain constant, then we have

$$E = E_0 + \sqrt{\frac{4E_g}{k}} \frac{1}{\sqrt{R_1}},$$

the rule established by the engineers. E_0, E_g and δ are obviously different for the two polarities of the wire. In favor of this theory is the phenomenon of beads. When a thin film of liquid is formed along a thin thread, the film on account of the surface tension breaks up into beads; similarly when a layer of electric energy is formed on the surface of the wire, it will have the tendency of breaking up into beads extending further away from the wire than the original layer. The fact that the negative discharge is much more apt to form beads than the positive one, seems to be connected with the mechanism of the discharge itself. When the wire is very thin negative electricity escapes easier than the positive one, just as in the case of very sharp points and at very low pressures.

The negative electricity seems to escape both from the molecules of the gas and of the metal, while the positive electricity consists only of positive ions, formed in the air alone, as no positive ions escape from the metal. The positive current consists of positive ions alone, the negative current of negative ions and electrons. Now it seems easier for the electrons to escape in a few places from the metal in large quantities, than from the entire surface of the wire in smaller quantities. That electrons escape from the neighborhood of the negative wire is also indicated by the fact that the negative wire bows in toward the positive one, which bows away from the negative one and that under the same circumstances the negative wire remains almost motionless while the same wire, when charged positively, carries out rotations of large amplitude.

For very small wires as well as for low pressures the negative corona starts before the positive one; for larger wires and higher pressures the positive corona starts before the negative. The negative electricity seems to escape in the form of electrons easier from thin metal wires than from molecules of the air. This phenomenon suggests that the average mass of the ions from small negative wires is smaller and the mobility larger than from larger negative wires.

Y. S. Townsend[2] has given another theory of the initial conditions of the glow discharge from wires, where he assumes the same values of the constants E_0 and b; and applies the same theory to the corona as to the spark discharge. The two phenomena are however in many respects different.

The law for ionization of a gas by collision can be expressed as follows:

$$\frac{\alpha}{p} = f\left(\frac{E}{p}\right).$$

Y. S. Townsend made an interesting application of this rule, based on experiments at low pressures, to the corona and spark discharge, which phenomena he considers as due entirely to the same process of ionization. Let us choose the following relations between the two cylinders in which

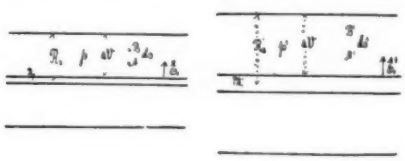

Fig. 1.

the corona occurs,

$$R_1' = zR_1,$$
$$R_2' = zR_2,$$

[1] Y. S. Townsend, The Electrician, June 6, 1913, p. 348.

$$p' = \frac{p}{z},$$

$$ds = zds',$$

z being a given constant number. $V = \Delta V$ is equal to the potential difference applied in both cases:

$$E_1 = \frac{V}{R_1 \log \frac{R_1}{R_1}}; \quad E_1' = \frac{V}{R_1' \log \frac{R_1'}{R_1'}} = \frac{V}{zR_1 \log \frac{R_1}{R_1}} = \frac{E_1}{z}.$$

$E_2 = E_1/z$ holds not only on the surface of the inner wire but in any two corresponding points such as A and A' or B and B'.

The number of ions formed by collision when a negative ion travels over the distance $AB = ds$ is given by:

$$\alpha ds = pdsf\left(\frac{E_1}{p}\right).$$

The number of ions produced in the second experiment over the distance ds' is given by:

$$\alpha' ds' = p'ds'f\left(\frac{E_1'}{p'}\right) = \frac{p}{z} zdsf\frac{\frac{E}{z}}{\frac{p}{z}},$$

$$\alpha' ds' = pdsf\left(\frac{E_1}{p}\right) = \alpha ds,$$

a negative ion traveling through the distance ds produces the same effect as over the distance ds'. The same holds for the collision of positive ions.

$$\beta ds = \beta' ds',$$

hence we have the same effects in both tubes. A given potential difference V causes the same phenomena in both tubes. If V is sufficient to start corona in one cylinder, it will also give rise to it in the other cylinder. If

$$R_2'p' = zR_2\frac{p}{z} = R_2p;$$

if

$$R_1'p' = R_1p$$

and if

$$V = V$$

then

$$R_1'E_1' = E_1R_1.$$

R_1p is therefore only a function of R_1E_1. If we keep R_1E_1 constant, R_1p remains constant.

This theory applies to the beginning spark as well as to the beginning glow discharge. It does not give an answer to one of the first questions regarding the corona discharge, namely, is the current due to ionization by negative or positive, or to both ions?

Now the following relation holds between the critical electric force E_1 and the radius R_1

$$E_1 = E_0 + \frac{b}{\sqrt{R_1}},$$

$$E_1R_1 = E_0R_1 + \frac{bR_1}{\sqrt{R_1}}, \qquad \text{for } p = 1,$$

or

$$E_1R_1 = E_0R_1 \cdot 1 \cdot + \frac{bR_1 \cdot 1 \cdot}{\sqrt{R_1 1 \cdot}}.$$

But if we keep $R_1 \cdot 1 \cdot = R_1p$ = constant, then E_1R_1 remains constant,

$$E_1R_1 = E_0R_1p + \frac{bR_1p}{\sqrt{R_1p}},$$

$$E_1 = E_0p + \frac{bp}{\sqrt{R_1p}},$$

$$E_1 = p\left(E_0 + \frac{b}{\sqrt{R_1p}}\right),$$

a rule that has been established by the engineers, before the theory was developed.

TABLE II.

p in Mm.	$+V$ in Volts.	$+E_1$ Volts per Cm. $\times 10^4$.	$+E_1$ Calcul.	$-V$	$-E_1$	$-E_1$ Calcul.
2.	720	0.765	0.33	580	0.615	0.33
10.9	940	0.998	0.80	870	0.925	0.81
18.9	1,110	1.18	1.07	1,200	1.275	1.08
53.2	1,770	1.88	1.88	1,920	2.04	1.94
91.3	2,350	2.50	2.56	2,580	2.74	2.64
173.5	3,450	3.60	3.72	3,750	3.99	3.86
248.5	4,250	4.51	4.65	4,610	4.90	4.84
334.8	5,120	5.42	5.58	5,520	5.86	5.86
483.6	6,660	7.08	7.11	7,120	7.55	7.45
616.6	7,800	8.29	8.33	8,330	8.85	8.77
720.0	8,730	9.28	9.21	9,210	9.80	9.72
746.0	8,980	9.51	9.51	9,530	10.1	10.1
768.3	9,100	9.67	9.65	9,640	10.2	10.2

Table II. gives as function of the pressure the electric force at the surface of the wire given by the potential difference V and the radii and cal-

culated by the last formula for the positive and negative wire. For the positive wire the rule holds from atmospheric pressure down to 53 mm. or lower, while for the negative wire the deviations are noticeable for much higher pressures. For the positive wire the constants $E_0 = 31.6$; $b = 8.43$ and for the negative wire $E_0 = 35.0$; $b = 8.06$ have been used, values that have been determined in a previous set of experiments.

If ionization by collision were due to negative ions alone, the constants E_0 and b would be the same; their difference indicates, that either negative and positive ions act as agents of ionization by collision or that another source of ionization is involved in the beginning corona.

It has been mentioned that the pressure of the gas suddenly increases, when the corona appears. We shall apply the principle of conservation of energy to this phenomenon as follows: Let the pressure of the gas in the corona tube be equal to p_0, let the potential difference applied be equal to ΔV, the current i and the volume v; the pressure will rise from p_0 to p_1; the work done by the current is equal to ΔVi and consists in the ionization of the gas plus the radiation of light U, and a reversible part W_1

$$\Delta Vi = U + W_1.$$

It is evident that if the current increases the current must decrease with increasing pressure, otherwise a finite amount of electric work $\Delta V \cdot i$ could perform an infinite amount of mechanical work.

After this first process we shall increase the pressure from the outside by dp_1 by means of the work

$$dW_2 = vdp_1,$$

so that the pressure rises from p_1 to $p_1 + dp_1$. Now we shall try to reach the same final state of the gas starting from the same initial conditions. Only this time we shall apply external pressure at first, supplying the work:

$$dW_3 = vdp_0,$$

while the pressure rises from p_0 to $p_0 + dp_0$. Now the same potential difference ΔV applied will produce a current i', and the work done by the current will be equal to:

$$\Delta Vi' = U' + W_4$$

and the pressure will rise to $p_1 + dp_1$. If the initial and the final conditions are the same, then the principle of conservation of energy leads to the following equation:

$$\Delta Vi - dW_2 = \Delta Vi' - dW_3,$$

$$U + W_1 - vdp_1 = U' + W_4 - vdp_0.$$

$$\Delta V(i - i') = v(dp_1 - dp_0).$$

But

$$i' = i - di,$$

$$\Delta V di = vd(p_1 - p_0)$$

$$i = \frac{v}{\Delta V}(p_1 - p_0) + i_0,$$

that is, the current will decrease with increasing pressure p_1, and hence the ionization pressure p_1 will increase proportionally to the current. Experiments will soon be published which will show that this conclusion holds in a wide range of currents and ionization pressures.

SUMMARY.

A description of the numerous phenomena has been given which are connected with direct current corona. The difference between positive and negative electricity appears in electrical, optical, mechanical and probably chemical effects. A new attempt at an explanation of some of the relations disclosed by experiments has been made. Relations between the critical electric force at the surface of the wire, the pressure of the air, and the radius of the wire have been obtained. The constants in these relations are different for the positive and the negative wire. The principle of conservation of energy has been applied to the ionization pressure and it is predicted that over a certain range the current should be directly proportional to the pressure increase.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
March, 1916.

THE EFFECT OF TEMPERATURE UPON THE ABSORPTION SPECTRUM OF A SYNTHETIC RUBY.

BY K. S. GIBSON.

INTRODUCTION.

THE absorption and fluorescence spectra of the ruby have been studied by Becquerel,[1] by du Bois and Elias,[2] and by Mendenhall and Wood.[3] There are in the spectra a number of lines[4] and narrow bands. The ordinary and extraordinary spectra differ somewhat and are quite simple compared with those of many other crystals. For these reasons, these investigators have considered the spectrum of the ruby as one of the best available for the study of the Zeeman-effect. While primarily engaged in the study of this and allied phenomena, they have also noted the change in the positions of the lines and bands which occurs upon a change in temperature.

The coefficient of absorption throughout the spectrum is also greatly influenced by temperature. In this paper are given the results of an investigation into the effect of temperature upon the coefficient of absorption of the ruby, as well as upon the positions of the various lines and bands in its absorption spectrum.

THE ABSORPTION CURVE.

For studying the changes in the absorption curve, a Lummer-Brodhun spectrophotometer was used. Light from an acetylene flame was passed through the ruby (which could be placed either in a furnace for high temperatures or in a Dewar flask for low temperatures), and was then reflected by a mirror into one of the collimators of the spectrophotometer where its intensity was compared in the usual way with the intensity of light from the same source reflected by another mirror into the other collimator of the instrument. Temperatures were measured by means of a thermo-junction and potentiometer. The apparatus was practically the same as was previously used in the study of the absorption curves of

[1] J. Becquerel, Compt. Rend., Vol. 151, p. 859; Vol. 151, p. 1344; Vol. 152, p. 183.

[2] H. du Bois and G. J. Elias, Ann. der Phys., Vol. 27, p. 233; Vol. 35, p. 617.

[3] C. E. Mendenhall and R. W. Wood, Phil. Mag., Vol. 30, p. 316.

[4] The word line, as used in this paper, refers to exceedingly narrow bands, which, in the spectrometer, appear to the eye as lines.

certain glass specimens, and is described in detail in a recent number of THE PHYSICAL REVIEW.[1]

The ruby was irregular in shape, and it was possible to send light through it in two directions. Unfortunately, however, the optic axis of the crystal was nearly at 45° with each of these directions; and for this reason it was found impossible to measure separately and satisfactorily the ordinary and extraordinary absorption curves. As the color of the extraordinary ray is red while the ordinary ray has a violet shade, decidedly different spectra could be expected. Even with this specimen there was a considerable difference in the shape of the ordinary and extraordinary absorption curves at room temperature; but the results were so unsatisfactory that they are not given, and no attempt was made to separate the two spectra at the other temperatures.

Much better results were obtained however in the study of the unpolarized spectrum. Even here trouble was experienced on account of the fact that the faces of the specimen were not quite parallel; and it was found that the intensity of the transmitted light when the ruby was placed in the furnace or in the Dewar flask was not as great as when placed at the same temperature directly in front of the slit of the spectrophotometer. There was however practically a constant ratio between the intensity of the transmitted light at various wave-lengths when the specimen was directly in front of the slit and when in the furnace, and also a constant though different ratio between the intensity when in front of the slit and when in the Dewar flask. Therefore all the values of the per cent. transmission obtained when the specimen was in the furnace or in the Dewar flask were changed by means of the proper ratio to correspond to the values obtained at 20° C. when the ruby was directly in front of the slit.

The values of the coefficient of absorption K were computed by the formula $K = (\log 1/P)/x$, where P represents the per cent. of the incident light transmitted (after correcting approximately for the light reflected at each surface) and x is the average thickness of the ruby, 2.9 mm. The values of $\log 1/P$ were obtained graphically from a curve plotted between values of P and $\log 1/P$.

The transmission and absorption curves are shown in Figs. 1 and 2. The temperatures at which observations were made are indicated in each figure. In order to avoid confusion, some of the values obtained for the per cent. transmission between .50 μ and .58 μ are not plotted in Fig. 1; and in Fig. 2, the curves for the temperatures $-$ 85°, 100°, and 320° C. are omitted. The amount of spectrum included by the slit-width used (.40 mm.) is indicated in each figure at .46, .56 and .66 μ.

[1] K. S. Gibson, PHYS. REV., N.S., Vol. VII., p. 194, 1916.

A number of interesting things may be seen from the curves. As the temperature is increased, the point of maximum transmission in the blue shifts from about .469 μ to .493 μ; and there is a shift of the absorption band in the green from about .548 μ to .566 μ (Fig. 2). It is

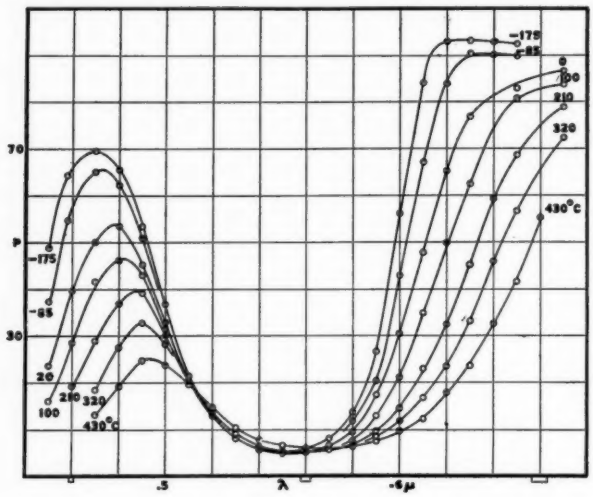

Fig. 1.

Variation of the per cent. transmission P with wave-length λ at different temperatures

interesting to see how much more the absorption changes with temperature on the red side of the band than on the blue side. This results in a broadening of the absorption band as the temperature is increased.

It will also be noticed how the transmission band in the blue is nearly cut out at the high temperatures on account of the violet side changing so much faster than the red side. As might be expected, this increased absorption in the blue results in a marked change of color. At 430° the ruby loses all of its rich violet shade and becomes a very light pink. Throughout most of the spectrum there is a large increase in the absorption as the temperature is raised, but in the blue-green the absorption is decreased. It should be remembered that these changes shown by the curves are the result of two separate changes, one in the ordinary and one in the extraordinary spectrum. The curves from .58 μ towards the red are very similar in shape to the curves obtained with the specimens of Cd Se glass studied by the writer, and the changes in absorption are of the same order of magnitude.

The Lines and Bands in the Absorption Spectrum.

For investigating the changes in the wave-lengths of the narrow absorption bands and lines, a Hilger constant-deviation spectrometer was

used. The ruby was placed in a furnace or in a Dewar flask, and light from a carbon arc was passed directly through the ruby and into the slit of the spectrometer. The apparatus was arranged so that readings could be taken of any line or band desired; then the furnace or Dewar flask containing the ruby was rotated out of the path of the light, and readings taken of the known lines of some other source of light; and then the ruby was brought back into position and the previous readings checked.

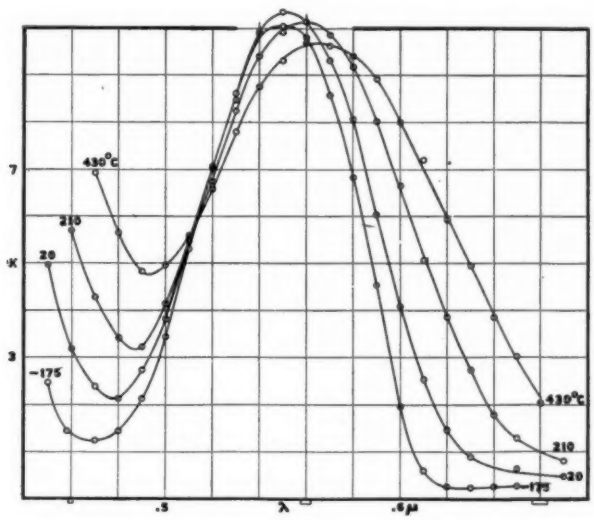

Fig. 2.

Variation of the coefficient of absorption K with wave-length λ at different temperatures.

The absorption lines and bands in the spectrum of the ruby at 20° C. are situated as follows, wave-lengths being given in Ångström units. In the red there are two heavy lines, one at 6943, R_1, and the other at at 6928, R_2; faint bands at 6688, R_3, and 6593, R_4; and in the blue, stronger bands at 4768, B_1, 4753, B_2, and 4687, B_3. In the case of the bands, the wave-length given is for the point of maximum absorption as estimated by the eye. Since the bands are very narrow, this could be done quite accurately.

The known wave-lengths used to determine the positions of R_1 and R_2 were the chromium lines 6978.9, 6924.9 and 6882.7, obtained by drilling a hole in the positive carbon of a carbon arc and filling with Cr_2O_3. The known wave-lengths used to determine R_3 and R_4, and the bands that appeared in the yellow upon cooling, were the lines from a Hg arc; and those used to locate B_1, B_2 and B_3 were the zinc lines 4810.7, 4722.3 and 4680.3, obtained by putting ZnO in a carbon arc.

From five to fifteen readings were always taken upon any one line or

band, both in the absorption and in the comparison spectra. The total variation among a given set of readings in the red was never more than five Ångström units and usually not more than three, while in the blue on account of the greater dispersion the variation was very seldom more than two units. The values as given for the lines and bands in the red part of the absorption spectrum are therefore probably accurate to within two or three Ångström units and those in the blue to within one Ångström unit.

In Table I. are given the wave-lengths of the various lines and bands as found at the temperatures indicated, and also the wave-lengths as given by other investigators.

TABLE I.

Number.	Character.	Temperature.	Wave-lengths.			
			Observed.	du Bois and Elias.	Wood.	Becquerel.
R_1.....	Line	−180° C.	6934	6932	6934	6932
	"	+ 20°	6943	6941	6946	
	"	210°	6956	(225°) 6960		
	"?	430°	6974			
R_2.....	Line	−180°	6919	6918	6920	6918
	"	+ 20°	6928	6926	6932	
	"	210°	6938	(225°) 6945		
	"?	430°	6956			
R_3.....	Band?	−180°	6681	6667, 6689		
	"	+ 20°	6688	6666–6701		
R_4.....	Band?	−180°	6585	6575–6598		
	"	+ 20°	6593	6575–6605		
Y_1.....	Band	−180°	5965	5960–5974		
Y_2.....	Band	−180°	5905	5881–5912		
B_1.....	Line	−180°	4763	4763		4763
	Band	+ 20°	4768	4769		
B_2	Line	−180°	4747	4746		4746
	Band	+ 20°	4753	4753		
B_3.....	Band	−180°	4681	4680		
	"	+ 20°	4687	4684		

As stated before, the wave-length given for the bands is for the point of maximum absorption as estimated by the eye. du Bois and Elias give the wave-length of the edges of most of the bands. At − 180° it was questionable whether R_3 and R_4 were bands or doublets. At 430° it was difficult to distinguish R_1 and R_2 as they are merging into a single band at that temperature. Bands Y_1 and Y_2 break off from the broad absorption band in the green as the temperature is lowered to − 180°. Bands B_1, B_2 and B_3 are very narrow at 20°. B_1 and B_2 become sharp heavy lines at − 180°, but B_3 is not greatly affected. At 210° the in-

crease in the general absorption in the blue, as shown in Figs. 1 and 2, made it impossible to get accurate readings on these bands.

In general, as can be seen from the table, the results obtained agree closely with those obtained by the other investigators, except that the large number of fine absorption lines between 6950 and 7100 which were found by Becquerel and by du Bois at − 190° could not be detected even through a thickness of 14.8 mm. There seems to be quite a discrepancy between the wave-lengths of R_1 and R_2 at 20° as given by du Bois and by Wood.

A detailed investigation into the positions and characteristics of the various lines and bands in both the ordinary and extraordinary spectra has been made by du Bois and Elias. They give colored maps which show very well the appearance of the absorption and fluorescence spectra in the red. The temperature was carried beyond 1000° C. R_1 and R_2 were found to merge into a single band at the higher temperatures.

In Table II. are given the values of the mean wave-length λ, *i. e.*, $(R_1 + R_2)/2$, at various temperatures T from − 190° to 845° C. as found by them.

TABLE II.

T	−190°	−79°	+18°	225°	300°	435°	540°	630°	845° C.
λ	6925	6930	6934	6952	6956	6969	6983	6995	7037

They mention the fact that these values show that the position of the lines changes faster than the temperature, but otherwise make no use of the data.

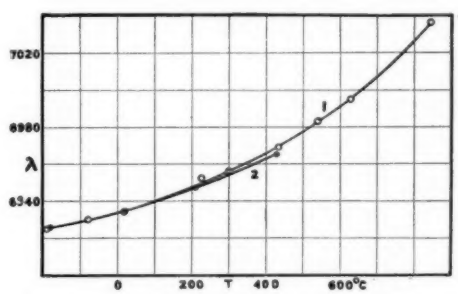

Fig. 3.

Variation of the mean wave-length λ of the absorption doublet, *i. e.*, $(R_1 + R_2)/2$, with temperature T. Curve 1: from values given by du Bois and Elias. Curve 2: as found by the writer.

In Fig. 3 these values are plotted as a curve; and in the same figure a second curve is drawn with the mean wave-length of R_1 and R_2 as found by the writer.

The equation of these curves is of the form,

$$\lambda = a + be^{kT}, \tag{1}$$

where T is the absolute temperature and a, b and k are constants. These constants may be obtained as follows. From equation (1),

$$\log (\lambda - a) = kT + \log b. \tag{2}$$

From this, by differentiation,

$$\frac{d\lambda}{dT} = k\lambda - ak. \tag{3}$$

Therefore if a curve be plotted between values of $d\lambda/dT$ and λ, it should be a straight line, whose slope is equal to k and whose x-intercept gives the value of a.

In Table III. are given values of the wave-length λ at every 100° and also of the slope $d\lambda/dT$ in Ångström units per degree as obtained graphically from the curves in Fig. 3.

TABLE III.

From Curve 1, Fig. 3.

T	−100°	0°	100°	200°	300°	400°	500°	600°	700°	800° C.
λ	6928.5	6933.4	6939.8	6947.2	6956.0	6965.8	6978.2	6990.8	7006.5	7025.5
$\frac{d\lambda}{dT}$	.0433	.0555	.0675	.0803	.0908	.1053	.1255	.1457	.1713	.2080

From Curve 2, Fig. 3.

T	−100°	0°	100°	200°	300°	400° C.
λ	6928.5	6933.4	6939.5	6946.0	6954.0	6962.5
$\frac{d\lambda}{dT}$	.0433	.0535	.0625	.0715	.0820	.0928

These values of $d\lambda/dT$ are plotted against λ in Fig. 4, and evidently lie along a straight line. The value of a seems to be exactly 6900 for both curves. Therefore equation (2) becomes,

$$\log (\lambda - 6900) = kT + \log b.$$

Now if the values of $\log (\lambda - 6900)$ be plotted against the corresponding temperatures, they should give a straight line whose slope is equal to k and whose y-intercept (*i. e.*, the value of $\log (\lambda - 6900)$ when $T = -273°$ C.) gives the value of $\log b$. In Table IV. are given the values of $\log (\lambda - 6900)$, as computed from the values of λ and T given by du Bois and Elias, and as found by the writer.

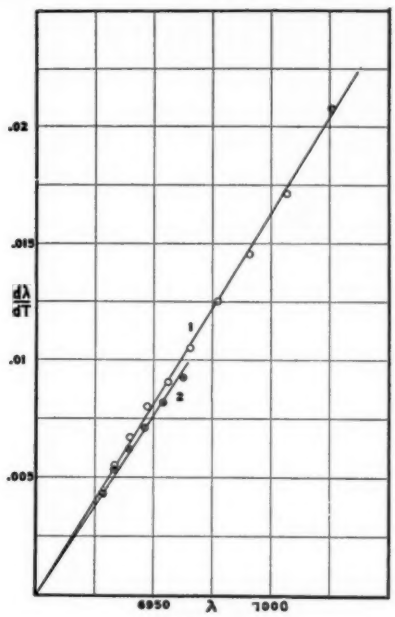

Fig. 4.

Variation of $d\lambda/dT$ with wave-length λ. Values of $d\lambda/dT$ obtained graphically from the curves in Fig. 3.

TABLE IV.

From Values of λ and T Given by du Bois and Elias.

T	−190°	−79°	+18°	225°	300°	435°	540°	630°	845°
λ	6925	6930	6934	6952	6956	6969	6983	6995	7037
log (λ−6900)	3.219	3.401	3.526	3.951	4.025	4.234	4.419	4.554	4.920

As Found by the Writer.

T	−180°	20°	210°	430°
λ	6926.5	6935.5	6947	6965
log (λ−6900)	3.277	3.569	3.850	4.147

In Fig. 5 these values of log ($\lambda - 6900$) are plotted against temperature and in each case lie along a straight line. From Curve 1, log $b = 3.082$ and $b = 22$. From Curve 2, $b = 23$. From Curve 1, Fig. 4, and Curve 1, Fig. 5, $k = .0163$. From Curve 2, Fig. 4, $k = .0154$, and from Curve 2, Fig. 5, $k = .0147$, or average $k = .015$.

Therefore equation (1) becomes, for Curve 1, Fig. 3,

$$\lambda = 6900 + 22e^{.0163T},$$

and for Curve 2, Fig. 3,

$$\lambda = 6900 + 23e^{.015T}.$$

It seems hardly probable that this perfectly definite relation between wave-length and temperature for this pair of absorption lines is accidental. Unfortunately the other lines and bands in the spectrum of the ruby are so broad and indefinite at temperatures much above 20° C.

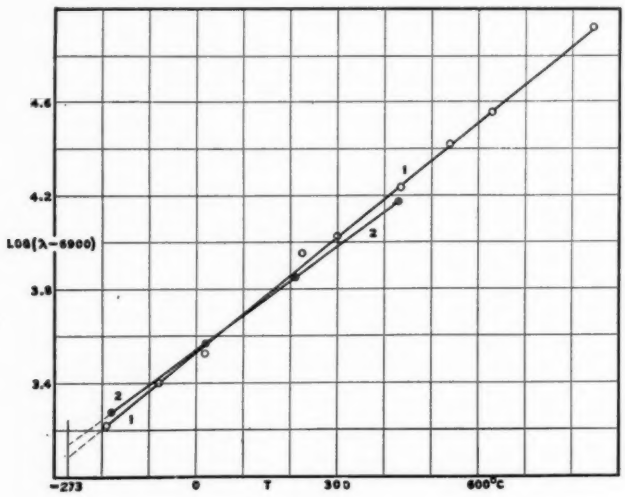

Fig. 5.

Variation of log (λ − 6900) with temperature T. Curve 1: from values of λ and T given by du Bois and Elias. Curve 2: from values found by the writer.

that accurate values of their wave-lengths cannot be determined. The shift of B_1 and B_2 between − 180° and 20° is fairly definite, but this variation in temperature is too small to be of any value in this connection.

It would be very interesting to know whether or not the lines in different parts of a spectrum obeyed this same general relation, and if so, how the constants varied. It would also be important to know whether the absorption lines in the spectra of other substances obeyed a similar relation; *i. e.*, whether or not the wave-lengths of any or all other absorption lines could be expressed as a similar function of the temperature.

As far as the writer has been able to discover, such a relation between wave-length and temperature has not been previously observed; and with the exception of the above values, taken from the work of du Bois and Elias, no data have been found which would be of value in testing out this relation.

These fluorescence and absorption lines are due to a slight admixture of Cr_2O_3 in the Al_2O_3 of which the ruby is made. It is thus probably more than a coincidence that the chromium spectrum, which was used as a

standard in determining the wave-lengths of these two absorption lines, has the three brilliant lines, previously mentioned, in the exact region of the spectrum in which these two absorption lines are situated. And the fact that these lines are due to the slight mixture of Cr_2O_3 in Al_2O_3 may furnish a clue as to the meaning of the relation found above. It is possible that the wave-length 6900, as found, is the mean wave-length of the two lines corresponding to the natural frequencies of the resonators of the active material; *i. e.*, the wave-length corresponding to the frequencies which these resonators would possess if uninfluenced by the surrounding molecules of Al_2O_3 composing the crystal. If that be true, the additional wave-length of 22 or 23 Ångström units at absolute zero might be considered as due to the decrease in frequency of the resonators, brought about by the influence of the surrounding material; and the factor e^{kT}, by which the 22 or 23 is multiplied, would indicate the effect of temperature in changing this influence. This of course is purely a guess, and no attempt has been made to go deeper into the theory of this relation until it is found whether or not a similar relation exists for absorption lines in other substances. It is hoped soon to make such an investigation.

SUMMARY.

1. The effect of temperature upon the absorption spectrum of a synthetic ruby has been studied, and curves drawn showing how the per cent. transmission and the coefficient of absorption vary with the wave-length at different temperatures.

2. The wave-lengths of the various lines and bands in the absorption spectrum have been determined. These values check closely with those obtained by other investigators.

3. A relation has been found between the wave-length λ and the absolute temperature T for an absorption doublet in the red part of the spectrum. This relation is of the form

$$\lambda = a + be^{kT},$$

where a, b and k are constants which may be determined graphically.

The writer wishes to express his gratitude to Professors Nichols, Merritt and Gibbs for their interest and for their many helpful suggestions in connection with these investigations.

CORNELL UNIVERSITY,
March, 1916.

A HIGH VACUUM MERCURY VAPOR PUMP OF EXTREME SPEED.

BY IRVING LANGMUIR.

ASPIRATORS or ejectors in which a blast of steam or air is used to produce a partial vacuum have been in use many years. For example the vacuum in the automatic vacuum brake system is obtained by such a device. The Parsons' vacuum augmenter used in the condensers of steam turbines produces a pressure as low as a few centimeters of mercury.

In these devices the high velocity of the jet of steam causes, according to hydrodynamical principles, a lowering of pressure, so that the air to be exhausted is sucked directly into the jet. If, however, the jet were surrounded by a perfect vacuum it can be readily seen from the principles of the kinetic theory that there would be a blast of gas molecules escaping from the jet into this vacuum. It is therefore not possible to obtain a very high vacuum by means of an instrument based on the principle of the ordinary ejector or aspirator.

The action of the aspirator or ejector, however, really consists of two processes.

1. The process by which the air is drawn into the jet.

2. The action of the jet in carrying the admixed air along into the condensing chamber.

The aspirators cease operating at low pressures because of the failure of the first of these processes. If air at low pressures could be made to enter the jet, and if gas escaping from the jet could be prevented from passing back into the vessel to be exhausted, then it should be possible to construct a jet pump which would operate even at the lowest pressures.

Gaede[1] has recently described a pump (called the diffusion pump) in which a blast of mercury vapor carries along the gas to be exhausted into the condenser in much the same way as the steam aspirator does (Process 2). In order to introduce the gas into the blast of mercury vapor (process 1), Gaede has used diffusion through a porous diaphragm, or through what amounts to the same thing, a slit of a width comparable with the mean free path of the mercury atoms in the blast. A portion of the mercury blast escapes through the slit, and the gas to be exhausted

[1] Ann. Phys., *46*, 357 (1915).

diffuses in against this blast of mercury vapor. This renders it necessary to make the slit very narrow (about 0.1 mm.) and for this reason the speed of the pump is necessarily relatively low.

Gaede defines the speed of a pump S by the equation

$$S = \frac{V}{t} \ln \frac{p_0}{p},$$

where V is the volume of the vessel being exhausted and t is the time required for the pressure to fall from p_0 to p.

Gaede gives some interesting data on the speed of his various pumps when exhausting air at about .001 mm. pressure.

Rotary mercury pump S = 120 cc. per sec.
Molecular pump S = 1,300 c.c. per sec.
Diffusion pump S = 80 c.c. per sec.

The great advantage of the diffusion pump is that the value of S remains constant down to the lowest pressures, whereas with all other forms of pump the value of S rapidly decreases when the pressure becomes much less than .001 mm.

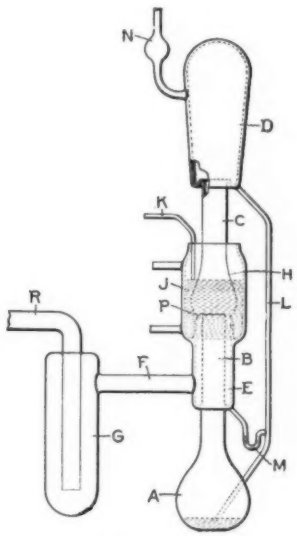

Fig. 1.

Some time ago it occurred to the writer that there should be other methods by which the gas to be exhausted may be introduced (Process 2) into the blast of mercury vapor. The serious limitation of speed imposed by the slowness of diffusion through narrow slits may thus be overcome.

Several methods have been found by which this may be accomplished. One of the types of pump which has given satisfactory results is that shown in the sketch (Fig. 1).[1]

[1] A preliminary announcement of the development of this pump was made April, 1916, by Miss Helen Hosmer (General Electric Review, *19*, 316, (1916)). Dr. H. B. Williams showed before the New York meeting of the American Physical Society (Feb. 26, 1916) a mercury vapor pump constructed entirely of glass which, unlike Gaede's pump, did not depend on diffusion through a narrow slit. This pump was subsequently described by Dr. Williams in an abstract in the PHYSICAL REVIEW, 7, 583, (1916). In regard to this abstract Dr. Williams has requested me to make the following announcement in his behalf:

"In describing recently before the Physical Society a mercury vapor pump, I stated that before carrying out the experiments I had in mind with a view to improving the Gaede type, I learned through Professor Pegram of Columbia that Dr. Irving Langmuir was already working with a pump in which the slit had been discarded and diffusion took place through a wide opening. I therefore constructed my own pump with a wide opening instead of experimenting with various widths as had been my original intention. In writing an abstract I omitted to make this acknowledgment owing to a desire to condense

In this device a blast of mercury vapor passes upward from the heated flask *A* through the tubes *B* and *C* into the condenser *D*. Surrounding *B* is an annular space *E* connecting through *F* and the trap *G* with the vessel to be exhausted. The tube *C* is enlarged into a bulb *H* just above the upper end of the tube *B*. This enlargement is surrounded by a water condenser *J* from which the water is removed at any desired height by means of the tube *K* which is connected to a water aspirator. The mercury condensing in *D* and *H* returns to the flask *A* by means of the tubes *L* and *M*. The tube *N* connects to the " rough " or " backing " pump which should maintain a pressure considerably lower than the vapor pressure of the mercury in *A*.

In this pump, the mercury atoms escaping from the upper end of the tube *B*, radiate out in all directions. A part of them passes up into *C*, but the larger part strikes the walls of the enlargement *H*.

If there is no water in the condenser *J* the mercury which condenses on the walls reëvaporates nearly as fast as it condenses. The molecules passing from the end of the tube *B* towards the walls *H* collide with the molecules which reëvaporate and may then be deflected downward into the annular space *E*. This blast of mercury vapor down through *E* prevents the gas from *F* from passing up into *H* so that under these conditions the gas from *F* may pass through the pump much more slowly than if no mercury vapor were produced in *A*.

On the other hand, when cold water circulates through the condenser *J*, *all* the mercury atoms striking the walls of *H* are condensed, so that no mercury passes down through *E*. The gas from *F* thus passes freely up through *E* and when it meets the mercury vapor blast at *P* is blown outward and upward along the walls of the condenser *H*, and finally forced into the main stream of mercury vapor passing up through *C* into the condenser *D*.

The main advantages of this pump are:

1. Simplicity and reliability.
2. Extremely high speed.
3. Absence of lower limits to which the pressure may be reduced.

The speed of a pump like that shown in the sketch has been determined for air and hydrogen.

A vessel of eleven liter capacity was connected to *R*. Air was admitted to the vessel, and the pressure was found to decrease as follows:

the abstract and because I expected to make such an acknowledgment later in print. The publication of a description of Dr. Langmuir's pump makes it seem desirable that this statement be made at the same time. H. B. Williams."

Time, Seconds.	Pressure in Bars.		Time Interval, Seconds.	Speed S Cc. per Sec.
	at R.	at D.		
0	1,470.	1,160		
30	294	720	> 30	590
60	12.8	218	> 30	1,150
80	0.015	18	> 20	3,700

The rough pump used had a speed of about 200 c.c. per second at pressures of 400 bars, but this speed fell off to 60 c.c. at 40 bars, and became zero at about 10 bars' pressure. The speed of the mercury vapor pump increased rapidly as the pressure decreased and reached a limit of about 4,000 c.c. per second at pressures below 10 bars. Theoretical considerations would indicate that this speed should remain constant down to the very lowest pressures.

Several experiments with hydrogen showed that the maximum speed with this gas was about 7,000 c.c. per second.

The pump shown in the sketch is made of glass. Many pumps operating on these principles have also been constructed of electrically welded sheet iron.

In a subsequent paper the writer will describe in more detail other modifications of mercury vapor pumps, some of which have marked advantages in simplicity of construction and in reliability of operation over that shown here. One particularly efficient type of pump is made wholly of metal.

The writer is greatly indebted to Dr. S. Dushman for assistance in the development of the new pump.

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AN EXPERIMENTAL VERIFICATION OF THE LAW OF VARIATION OF MASS WITH VELOCITY FOR CATHODE RAYS.

BY LLOYD T. JONES.

INTRODUCTION.

THE mass of the electron has been shown by W. Kaufmann[1] to be electromagnetic in origin. This mass is shown by the elementary laws of electromagnetism to be constant for small velocities of the electron. M. Abraham[2] has developed an electro-dynamic theory of moving electrons by which he accounts for the falling off of the ratio e/m for electrons moving with high velocities. If β is the ratio of the velocity of the electron to that of light, the ratio of the mass of the electron moving with the velocity v to its mass, m_0, when moving with a slow velocity is

$$\frac{m}{m_0} = \frac{3}{4\beta^2}\left[\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right].$$

The Lorentz-Einstein formula, which satisfies the principle of relativity, gives the ratio of the masses as

$$\frac{m}{m_0} = [1-\beta^2]^{-\frac{1}{2}}.$$

Kunz[3] has discussed the bearing of these formulæ in connection with an electromagnetic emission theory of light and has developed three forms of the formula based on possible changes of form of the electron.

Stark[4] has found that the mass of the cathode particle increases as the velocity increases. The maximum velocity employed by him, 1.14×10^{10} cm. per sec., was, however, not great enough to cause more than a small per cent. increase in the mass.

Later Guye and Ratnsoky[5] carried out an experiment employing rays of 14.7×10^{10} cm. per sec. velocity and obtained an increase of nearly twenty per cent. in the mass.

Each of the investigators has employed a method in which the cathode

[1] W. Kaufmann, Gott. Nachr., 1901, Heft 2; 1902, Heft 5; Phys. Zeitschr., 4, 54, 1902.
[2] M. Abraham, Gott. Nachr., 1902, Heft. 1.
[3] J. Kunz, Arch. des Sci., Jan. 1913; PHYS. REV., p. 464, 1914.
[4] H. Stark, Verh. d. Deut. Phys. Gesell., 5, p. 241, 1903.
[5] Guye and Ratnosky, Arch. des Sci., 31, p. 293, 1911. Guye and Lavanchy, Comptes Rendus, p. 52, July, 1915.

beam traverses nonuniform electric and magnetic fields and the deflection is shown on a phosphorescent screen placed perpendicular to the path. This necessitated a homogeneous cathode beam.

The conclusions of the experimenter must be based on a large number of observations taken at each of a number of different velocities. The method that has been developed for the present research lessens materially the difficulties encountered in a verification by cathode rays and is applicable equally well for the β particles of radium. Perhaps the best feature of the method is that it is desired to have rays of all possible velocities present in the discharge rather than a homogeneous beam. This allows one to use the discharge from a high potential transformer without any additional pieces of apparatus to operate during the time of exposure. Since from a single photograph calculations may be made of e/m for all the velocities present it is possible to obtain the desired results by a single exposure.

THE APPARATUS.

In a previous determination of e/m and v for cathode rays[1] an apparatus was used involving the same principles as this; the high discharge potentials used in the present investigation, however, necessitated a change in the manner of introducing the electrodes and more effective insulation guarding against the ionizing and direct effect of the discharge.

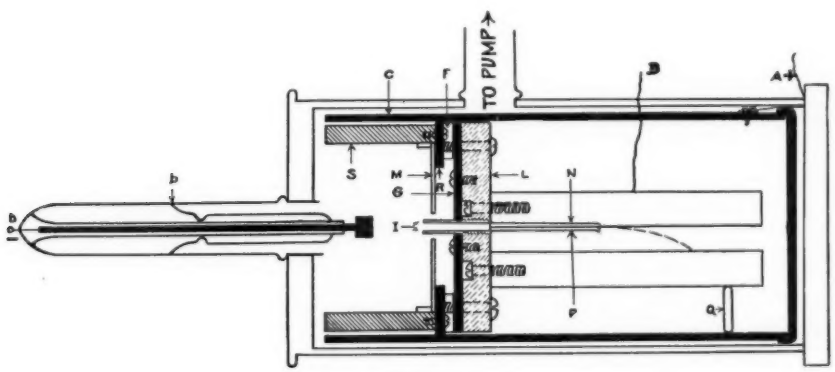


Fig. 1.

A glass jar 11.5 cm. in diameter and about 35 cm. long had a 2.2 cm. hole bored in its base through which the cathode was introduced. The cathode was an aluminum disc about .8 cm. in diameter carried on a small brass rod encased in a small glass tube and connected with one terminal of the transformer through a platinum wire sealed in the glass.

[1] L. T. Jones, PHYS. REV., N.S., Vol. III., p. 317, 1914.

The glass tube encasing the cathode rod was supported at two places by a second glass tube sealed, as shown at *b* in Fig. 1, to a tube of 2 cm. diameter which passed through the hole in the base of the jar. This tube was fastened to the base by sealing wax. A brass cylinder, *C*, of 10.2 cm. diameter and about 32 cm. long was fastened rigidly (manner not shown) to the glass jar; and a brass ring, *R*, of 1.5 cm. width and .4 cm. thickness was soldered inside it with its plane perpendicular to the axis of the brass cylinder. An iron cylinder, *S*, of 7.5 cm. inside diameter and .8 cm. thickness was fastened by screws to the ring *R*. An ebonite ring, *F*, of nearly the same dimensions as *R* was fastened to *R* by screws whose heads were sunk well beneath the surface next *G*. *G* was a brass disc of .3 cm. thickness fastened by brass screws to the ebonite disc, *L*, which was of .8 cm. thickness and carried the electrostatic plates. To increase the insulation discs of mica were placed between *G* and *L*, *F* and *G* and *R* and *F*. To prevent trouble due to the heavy discharge a brass disc, *M*, was held against *R* by a slip ring in *S*. A few millimeters' space was left around the small iron tube, *I*, which passed through directly in front of the cathode.

The two electrostatic plates were brass plates $20.5 \times 7.5 \times 1.2$ cm. In the upper one was inlaid a piece of soft iron, *N*, $5.13 \times 1.4 \times .1$ cm. A similar piece of iron, *P*, was held against *N* by eight short iron screws. After the iron piece, *N*, was inlaid and all necessary holes had been made in the plates they were annealed and then one side of each was surfaced to within .001 cm. of plane. The slip *P* also had its face next *N* made plane. A scratch of about .05 cm. width was drawn full length on the plane side of *P*. The ends of this scratch, for about 1 mm. of their length, were closed with solder and the solder cut off flush with the surface. A small cut was then made in each of the bits of solder and these cuts determined the path of the electron immediately before its entrance into the deflecting fields. The electron then takes the path indicated by the dotted line in Fig. 1. The electron is thus protected from the fields until it leaves the constricting canal. Care was taken that the small cut marking the entrance of the electron in the fields was perfect to the ends of *N* and *P* and that the ends of *N* and *P* were exactly even. As a final precaution a small bit of solder was placed in the middle of the canal as well and a small cut made in it. This insured a straight beam through the tube. Each of these cuts was .01 cm. in width and of about the same depth. The softest iron obtainable was used throughout and the brass was free of magnetic material.

The ebonite disc, *L*, with its plate, *G*, was held against the ring, *F*, by four heavy brass screws threaded into *R*. They were insulated from *G* by an air space of about 2 mm.

The two electrostatic plates were held at a fixed distance apart by four porcelain blocks placed one at each corner. Under the back end of the lower plate was placed a brass leg, Q, of adjustable height which served as an additional support for the plates and at the same time connected the lower plate electrically with the brass cylinder, and through it with M and S.

A hole 2.2 cm. in diameter was bored in the side of the glass jar at a suitable position and a glass tube waxed to the jar here connected the vessel to the molecular pump. A wire, A, was connected to the brass cylinder near the back where it could be easily reached from the outside. A was connected to earth and to the second terminal of the transformer. The surfaces of M and S served as anode. The upper of the electrostatic plates was connected electrically to the outside by a wire, B, passing through an insulating plug in the brass cylinder and through a small hole in the glass cylinder. The hole was made vacuum tight by sealing wax.

The photographic chamber was made light tight by closing the ends of the cylinder with a brass cap and the jar was made vacuum tight by closing with a glass plate sealed on with a mixture of beeswax and resin.

The electrostatic potential was applied to A and B and the transformer connected to A and D.

The Spacing Blocks.

Each of the four spacing blocks placed between the corners of the electrostatic plates was a length of a porcelain tube of 1.2 cm. external diameter. After the sections had been cut from the tube they were waxed inside a short piece of brass tubing whose outside was accurately round so it could be chucked in the grinding machine. They were ground down till they were of nearly the same length and the end planes parallel. They were then finished by hand on an iron plate with emery till they were very accurately the same length and the end planes parallel as the measurements showed. The length of the blocks was measured by an optical lever of 24.415 cm. length with a scale 3 meters distant. The lengths of the blocks were $.9822 \pm .0008$ cm.

The Electrostatic Potential.

A high potential storage battery, T, was used to send a small current through two high resistances, M and R, shown in Fig. 2. M consisted of two Wolff boxes aggregating 2×10^6 ohms and R was an adjustable resistance of 10^4 ohms. The potential drop across a part of M was compared by means of the potentiometer, P, with a Weston standard cell of 1.0183 volts at 24° C. The standard cell checked with one recently received

from the Bureau of Standards. By adjusting R the value was easily kept constant to within 1 volt and the value thus determined to less than .1 per cent. The two electrostatic plates were connected directly to the terminals of M.

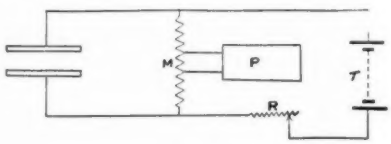


Fig. 2.

THE MAGNETIC FIELD.

The magnetic field was due to a constant current through 240 turns of wire wound on a rectangular wooden frame about 160 × 60 cm. The cross-section of the coil of wire was about 2 × 2 cm. Calculation showed the field to be uniform over a range greater than that used.

The field was calibrated by the aid of a solenoid of 1,141 turns and 149.83 cm. length wound uniformly on a wooden frame of about 6 × 9 cm. cross-section. The solenoid was placed in the geometrical center of the rectangular frame so that the fields either coincided or opposed each other. A small coil of about 200 turns of very fine wire wound on an ebonite rectangle 2 × 8 cm. was then placed in the center of the solenoid. This coil was connected to a Grassot fluxmeter whose scale was about 4 meters distant. A known constant current was sent through the coil to be calibrated and the current through the solenoid adjusted until the fluxmeter showed no deflection when the two currents were broken simultaneously. The ratio of the currents, 70 to 13.55, gave the value of the field to be 1.854 gausses per ampere. A field of .002 gauss produced a deflection of .3 mm. on the fluxmeter scale.

THE MEASUREMENT OF THE CURRENT.

The current in the magnetic field was measured with a Siemens & Halske ammeter of 150 scale divisions which, with the shunt used, had a range of 0 to 3 amperes. The ammeter was calibrated by passing a current through it in series with two Hartmann & Braun standard resistances of .1 and 1 ohm. The potential drop across each resistance was measured by the Wolff potentiometer against the Weston standard cell and the current calculated. The standard resistances were kept in an oil bath at constant temperature. The Reichsanstalt certificates showed the resistances to be sufficiently correct. The calibrations by the two resistances checked. Throughout the calibrations and experiment an adjustable resistance was used to set the ammeter needle exactly on a

scale mark in order that any variation in the current could be more easily detected. With special care taken for good contacts little difficulty was experienced in keeping the ammeter needle exactly on the division mark.

The Vacuum.

The Gaede molecular pump, with the Gaede mercury pump as a preliminary, was used for the exhaustion. The molecular pump was connected by 30 mm. tubing directly to the vessel to be exhausted with no stopcock or other constriction intervening. The mercury pump was connected to a McLeod gauge and a large tube of cocoanut charcoal. The order of starting the pumps assures freedom of mercury vapor in the discharge tube. The construction of the apparatus with its sealing-wax joints made it quite impossible to heat the vessel to rid it of moisture. Such a proceeding proved unnecessary with the wide connecting tubes used, however, as an hour of pumping was usually sufficient to produce a vacuum that caused the transformer to spark 20 cm. between its point terminals rather than pass through the discharge tube. This degree of rarefaction was usually produced without the aid of liquid air on the charcoal. To be sure the equivalent spark length of the tube always dropped a few centimeters during the time of discharge, but a half or at most one minute of pumping was sufficient to restore the vacuum.

It may be of interest to some users of the molecular pump to know that considerable trouble was experienced with the pump due to the creeping in of oil from the bearings. Once in about six weeks the pump became stiff and the half H. P. motor was unable to drive it at the normal speed used, 8,000 R. P. M. The oil was then taken from the bearings and the whole pump thoroughly washed with filtered gasoline and dried by drawing air through it. This operation usually required three days.

The Electric Discharge.

The transformer used to produce the cathode beam was one built for the ratio 110–40,000 volts operating on a 60-cycle circuit. For a number of photographs it was used on a 440-volt 60-cycle circuit. The rays thus produced were of rather a slow velocity although the vacuum was so high that the transformer sparked across a 20 cm. gap between points outside. In order to lessen the amount of energy used and still retain the potential the transformer was operated on 110-volts D.C. with a Wehnelt interrupter. This arrangement, with or without a capacity across the interrupter, gave rays of a much higher velocity. Under these conditions, however, the equivalent spark gap of the vacuum was only about 8 to 12 cm.

THE FORMULA.

The beam passes through uniform electrostatic and magnetic fields, whose lines are parallel to each other, and strikes the photographic plate which is lying on the lower electrostatic plate.

Let the particle be subjected to the simultaneous action of the electric and magnetic fields. The particle will be bent downward by the electric field and strike the photographic plate at a distance l (measured along the direction of the undeflected beam) from the source. It will at the same time be bent aside by the magnetic field a distance z (measured at right angles to l). Since many velocities are present they will show themselves in a long trace on the photographic plate and e/m may be calculated for any point in the trace and hence for that velocity. If the electrostatic field alone acts the resultant trace will be straight down the center of the plate. If the magnetic field also acts, then for each value of the current a trace will appear at the side and, when the current is reversed, a similar trace on the opposite side and at nearly the same distance from the center one. In photograph 58, Plate I., two values of the current were used which, with the central magnetically undeflected exposure, make five traces on the plate. The magnetic deflection, z, was taken as half the distance between two corresponding points of corresponding traces. The electrostatic plates were mounted horizontally. Each particle then describes an arc of a parabola in the vertical plane and an arc of a circle in the horizontal plane.

THE ELECTROSTATIC DEFLECTION.

Let d be the distance from the upper electrostatic plate to the upper surface of the photographic plate and let t be the thickness of the photographic plate, Fig. 3. If K is the dielectric constant of the photographic

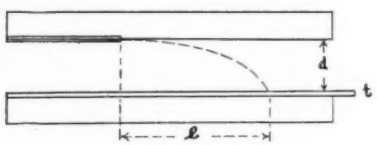


Fig. 3.

plate and V the potential difference in volts of the two electrostatic plates the force on unit charge due to the electric field is

$$E = \frac{V \times 10^8}{d + t/K}. \qquad (1)$$

This force produces a downward acceleration of the electron such that

$$Ee = ma, \qquad (2)$$

where e is the charge, m the mass and a the acceleration of the electron. If t_1 is the time required for the electron to fall to the photographic plate we shall have

$$vt_1 = \sqrt{l^2 + z^2}, \tag{3}$$

where v is the velocity of the electron. Equation (3) is true, since z is small enough in comparison with l that the chord may be considered equal in length to the arc. Then

$$t_1^2 = \frac{l^2 + z^2}{v^2}, \tag{4}$$

and, since

$$d = \tfrac{1}{2}at_1^2, \tag{5}$$

we get

$$d = \frac{Ee}{2m}\,\frac{l^2 + z^2}{v^2}. \tag{6}$$

Substituting the value of E from equation (1) and rearranging we have

$$\frac{mv^2}{e} = \frac{V(l^2 + z^2)10^8}{2d(d + t/K)}. \tag{7}$$

THE MAGNETIC DEFLECTION.

If the plane of the photographic plate be considered as in the plane of this page with the source of cathode rays at the origin, O, of the set of axes shown in Fig. 4, the arc of the circle shown will be the projection on the photographic plate of the electron's path and will show accurately the curvature of the path due to the magnetic field. Let z be the magnetic deflections, measured as previously mentioned, and let l again be the x distance to where some electron of velocity v strikes the plate. If r is the radius of curvature of the circular path due to the magnetic field, the length of the projection on the photographic plate of the actual path, or the arc shown, will be

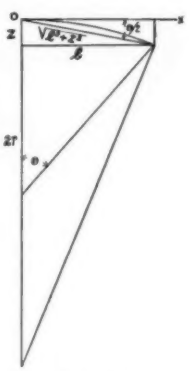


Fig. 4.

$$r\theta = vt_1, \tag{8}$$

where θ is the angle at the center of the circle subtended by the arc.

From Fig. 4 we see that

$$\tan\frac{\theta}{2} = \frac{z}{l} \tag{9}$$

and that

$$\tan\theta = \frac{l}{r - z}. \tag{10}$$

But

$$\tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \tag{11}$$

and from Fig. 4

$$\cos \theta = \frac{r - z}{r}. \tag{12}$$

Then

$$\frac{z}{l} = \sqrt{\frac{z}{2r - z}} \tag{13}$$

and

$$\frac{z^2}{l^2} = \frac{z}{2r - z}, \tag{14}$$

whence

$$r = \frac{l^2 + z^2}{2z}. \tag{15}$$

The magnetic force on the particle, due to the field H, is perpendicular to the direction of motion of the particle and hence has only its component, $Hev \cos \theta$, in the y direction. Since only this component produces the deflection z, we shall find the average force, $\bar{f}$, on the particle and use this value for the magnetic force.

$$\bar{f} = \frac{Hev \int_0^\theta \cos \theta d\theta}{\theta} = Hev \frac{\sin \theta}{\theta}. \tag{16}$$

This force gives the electron an acceleration a_1 in the y direction. Then

$$\bar{f} = ma_1 = Hev \frac{\sin \theta}{\theta}, \tag{17}$$

and

$$z = \tfrac{1}{2} a_1 t_1^2. \tag{18}$$

From equations (8) and (18) we have

$$t_1^2 = \frac{r^2 \theta^2}{v^2}, \tag{19}$$

and from (17)

$$a_1 = \frac{e}{m} Hv \frac{\sin \theta}{\theta}. \tag{20}$$

On substitution of the values from (19) and (20) in equation (18) we get

$$z = \frac{e}{m} \frac{Hr^2 \theta \sin \theta}{2v}. \tag{21}$$

From Fig. 4

$$\sin \theta = \frac{l}{r}, \tag{22}$$

and, with an approximation,

$$r\theta = vt_1 = \sqrt{l^2 + z^2}. \tag{23}$$

Substituting these values in (21) we find

$$z = \frac{e}{mv}\frac{H}{2}\sqrt{l^2 + z^2} \tag{24}$$

or

$$\frac{e}{mv} = \frac{2z}{H\sqrt{l^2 + z^2}}. \tag{25}$$

From equations (7) and (25) we obtain the desired expressions,

$$v = \frac{zV\sqrt{l^2 + z^2} \times 10^8}{Hld(d + t/K)} \tag{26}$$

and

$$\frac{e}{m} = \frac{z^2V2 \times 10^8}{H^2l^2d(d + t/K)}. \tag{27}$$

For any single photograph taken with constant deflecting fields equation (27) may be written in the form

$$z = C\sqrt{e/m}, \tag{28}$$

where C is a constant.

This equation shows the traces to be straight lines for constant values of e/m and that the outer traces should curve toward the central one for the higher velocities. From the way e/m enters the equation one would expect only slight curvature of the traces unless e/m diminished very rapidly. The equation shows that only the ratio z/l or the slope of the straight lines need be obtained from the photographic plates. This method is thus made one of particular value for the determination of e/m_0, for slow velocities, as it permits easy averaging of values.

The Earth's Field.

In fastening the apparatus to the stone pier it was carefully placed so that the undeflected beam travelled horizontally in the direction of the earth's field. The effect of the vertical component of the earth's field was then to increase the one deflection of the magnetic field and to lessen the deflection when the current was reversed. From an inspection of the equation it is seen that the effect of this vertical component may be neglected as it cancels due to the method used in measuring z.

The beam of electrons travelled from north to south so that when only the electrostatic field bends it downward it cuts the horizontal component of the earth's field at a small angle. The central trace is thus thrown a little to one side.

Let H_1 be the value of the horizontal component of the earth's field.

When the electron is deflected magnetically it has a component velocity at right angles to the magnetic field H_1 and therefore has a small force acting on it. This force will aid or oppose the force of the electrostatic field depending on the direction of the magnetic deflection. This small force due to H_1 was found to be negligible.

It follows then that a small error made in placing the apparatus such that the beam would travel neither quite horizontally nor exactly in the magnetic meridian would have no appreciable effect on the results of the experiment.

The dielectric constant, K, of the photographic plate was taken as 6. Since the plate is in contact with the lower electrostatic plate and the electrostatic field is on for ten to thirty minutes before the exposures are made the value of the constant chosen must not be that obtained by a method not allowing for the accumulation of a charge by the glass. It should be pointed out, however, that if the value of e/m_0 is measured from the same photograph the value of K will in no wise affect the value of the ratio m/m_0 if only K remain constant. It will enter, however, in the determination of the velocity of the electron but the error thus introduced is relatively small.

The deflecting magnetic field was kept at values sufficiently small that z^2 could be neglected compared with l^2. The equation for the velocity then becomes

$$v = \frac{zV \times 10^8}{Hd(d + t/K)}. \tag{29}$$

If the value of e/m is calculated from the smaller deflections, z_0 and l_0, on a photograph the ratio m/m_0 for the higher velocities is given by

$$\frac{m}{m_0} = \frac{z_0^2}{l_0^2}\frac{l^2}{z^2}. \tag{30}$$

The individual values of e/m as calculated from the photographs are shown in Fig. 5, in which (as well as in Figs. 6, 7 and 8) the full line curves marked "A" and "L" correspond to the theoretical values of Abraham and Lorentz respectively. Of the points lying above both of these curves all except three are due to a single photograph.

The ratio of the masses was also calculated by means of the preceding equation. To test which of the theoretical curves the points collectively best fit it was assumed that the value for the slowest velocity electrons showing on each of the photographs was a value exactly fitting the Lorentz curve and the other values were plotted by using only the ratio of the masses as calculated from the photographs. These values are set down in Fig. 6. Similarly Fig. 7 shows the results assuming the

value for the slowest velocity showing on each of the photographs to lie exactly on the Abraham curve. Now by a comparison of Figs. 6 and 7 it is seen that in either case the points fit the Lorentz curve more nearly than the Abraham curve.

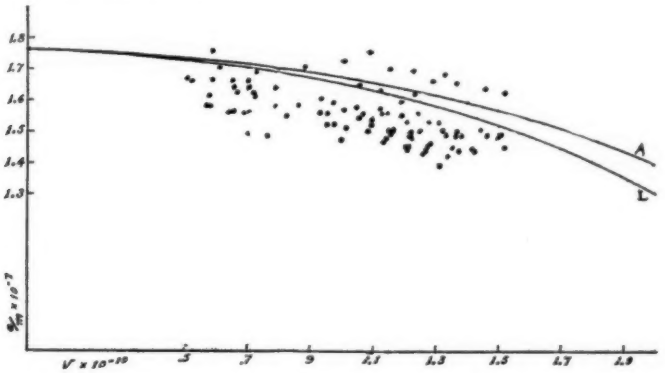


Fig. 5.

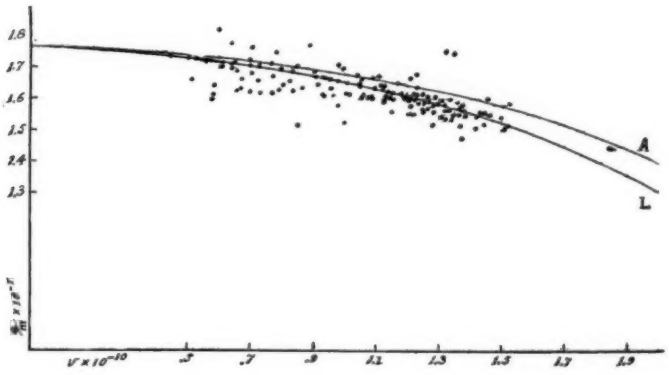


Fig. 6.

Table I. gives the data and results taken from one pair of traces on one of the photographs.

TABLE I.

l Cm.	$2x$ Cm.	x/l Cm.	$e/m \times 10^{-7}$.	$v \times 10^{-10}$.	Remarks.
3.93	.1443	.01836	1.708	.6089	
4.43	.1587	.01791	1.625	.6696	
6.93	.2503	.01806	1.653	1.056	Photograph 58,
7.43	.2667	.01795	1.632	1.125	1,926 volts,
7.93	.2817	.01796	1.599	1.189	$d=.8137$ cm.,
8.43	.2957	.01754	1.558	1.247	$d+t/K=.8418$ cm.
8.93	.3107	.01740	1.533	1.311	
9.43	.3250	.01723	1.504	1.371	
9.93	.3427	.01726	1.508	1.446	
10.43	.3583	.01718	1.495	1.512	

Fig. 8 represents graphically the results shown in Table I.

On each of the photographs the lines seen crossing the electron paths were drawn between the jaws of a pair of vernier calipers. The photographic plate while in position touched the ebonite disc, L, Fig. 1, and hence the length of the iron slip, P, determined the distance of the opening

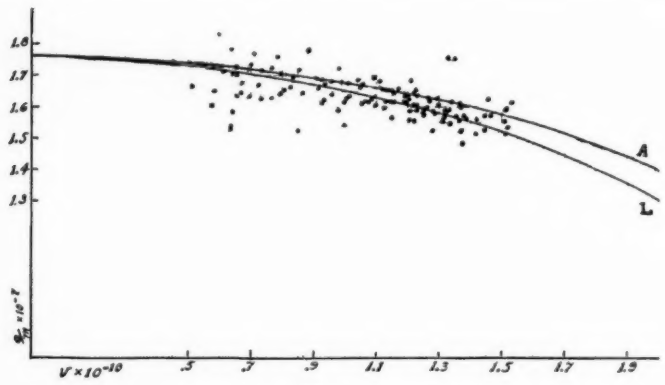


Fig. 7.

from the end of the photographic plate. On each of the photographs the line near the right is that marking the opening and the others show the successive values of l for which the values of e/m and v were calculated.

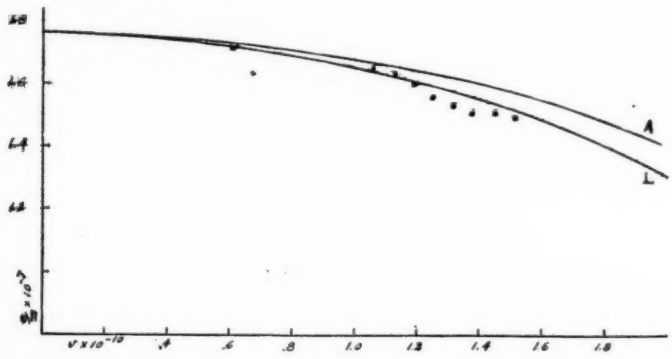
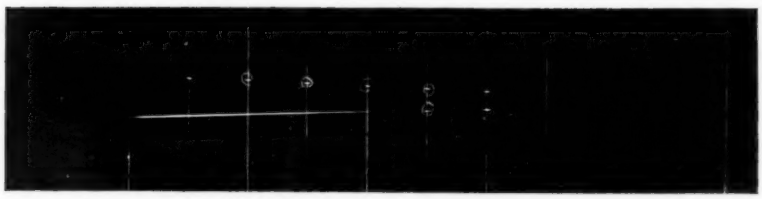


Fig. 8.

In several of the photographs, 54 for instance, the lines could be seen nicely with the unaided eye but were too dim when seen through the comparator microscope. These lines were touched with a sharp pencil and these marks used to determine the position of the lines. The photographs show very easily which of the lines were so treated.

The distance apart of the traces, $2z$, was measured by a comparator reading to .005 mm. The value of $2z$ for each distance was measured



Photograph 54.

Photograph 58.

Photograph 59.

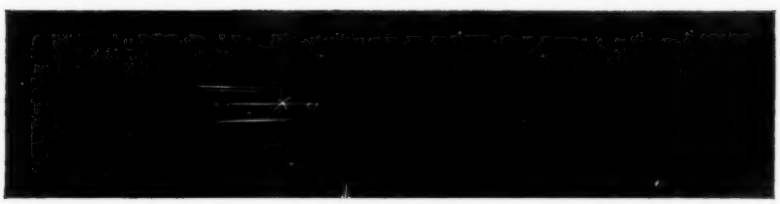
Photograph 60.

LLOYD T. JONES.

Photograph 63.

Photograph 64.

Photograph 66.



Photograph 68.

LLOYD T. JONES.

five times, usually on different days, and the average taken. Usually no measurement differed more than .001 cm. from the average and almost never did one differ more than .002 cm. The comparator screw was calibrated.

It was found experimentally that the distortion of the magnetic field due to the iron of the constricting canal had the effect of making the magnetic deflection smaller. It may be considered as zero for a small distance p further and then constant for the remainder of the path. This has the effect of making l smaller and hence e/m and v larger. The length l does not enter directly into the value of v unless the two values of l for the distance of travel in the two fields, electrostatic and magnetic, are different. The value 1.765×10^7 was assumed as the correct value of e/m_0 and the two curves in Figs. 5, 6 and 7 accordingly point to this value for slow velocities. The magnitude of the factor p was calculated and this value, $p = .07$ cm., was used to correct all the values of l used. This correction was assumed to be the same for the electrostatic field.

CONCLUSIONS.

1. The method used in the present investigation does not necessitate a homogeneous cathode beam.

2. Each photograph gives a trace of all velocities present and makes possible a verification of the law from a single photograph.

3. The cathode beam never leaves the region between the electrostatic plates. The uncertain field distribution at the ends of the plates is thus avoided.

4. The present investigation has been carried out with rays of a velocity v little greater than any previously employed.

5. The results favor the Lorentz-Einstein rather than the Abraham formula.

In conclusion I wish to express my appreciation to Dr. C. T. Knipp for the enthusiasm with which he has followed the progress of the work. Also I wish to express my thanks to Prof. A. P. Carman, director of the laboratory, for the facilities he so kindly placed at my disposal.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
March 1, 1916.

NOTES ON PAGE'S THEORY OF HEAT RADIATION.

BY DAVID L. WEBSTER.

THE paper referred to in the above title was published by Page in the PHYSICAL REVIEW of February, 1916, obtaining Planck's radiation formula by a method consistent with the classical electrodynamics and not involving so many radical assumptions as Planck's theory. Having made a similar attempt myself in 1915,[1] I am interested to see that Page and I were led independently to what are essentially the same fundamental assumptions. Nevertheless there are certain differences that seem to make my theory more consistent with phenomena. Moreover, while Page's theory seems to involve fewer assumptions, it appears on closer examination to imply one that looks less reasonable than those it eliminates.

The fundamental assumptions common to both theories are (1) that the connection between electricity and the ether is that of the classical electrodynamics, (2) that the oscillator contains a rotational degree of freedom storing energy in a non-radiating form, and (3) that under the proper conditions energy can be transferred by an internal mechanism between the radiating and non-radiating degrees of freedom.

In my theory this oscillator is an electron, assumed to have the properties ascribed to electrons by Parson in his "Magneton Theory of the Structure of the Atom,"[2] developed for the explanation of chemical and magnetic phenomena. According to Parson the electron, or magneton, as he calls it, is a very thin ring of negative electricity something like 1.5×10^{-9} cm. in radius, revolving with a velocity of the order of that of light, and therefore having magnetic, as well as electrostatic, properties.

The magneton thus conceived as a continuous ring rotating in its own plane has energy stored in a non-radiating form. If the electricity is movable on the ring in any other way than as a rigid mass, the alternating external force of a light wave will induce oscillations on it capable of absorption and radiation of energy. These induced oscillations were shown in my paper to give an explanation of refraction, diffraction and allied phenomena almost exactly like that of the classical electron theory. At the same time the internal mechanism assumed above for

[1] Amer. Acad. Proc., Jan., 1915.

[2] Smithsonian Miscellaneous Collections, Volume 65, No. 11, 1915.

transferring energy between the oscillations and the circulation can be supposed to damp the induced oscillations, transferring their energy to the rotation, until the energy accumulated above the initial value reaches some definite multiple of the quantum. At such a point it is assumed that the process may be reversed, starting a larger oscillation, which will be maintained constant at the expense of the circulation, until the excess energy is all radiated away. The probability, η, that such an oscillation will start when the stored energy reaches a given multiple of $h\nu$ is given by Planck's condition $(1 - \eta)/\eta = pI$, where I is the mean square of the electric field per unit frequency interval, and p is determined so as to give the Jeans-Rayleigh law at low frequencies. The entropy of the system is then found by Planck's equation $S = k \log W$, where the microscopic state of a system is determined by the total accumulated energy of each oscillator. The derivation of Planck's law in this theory is therefore almost exactly like his. The advantage of this system is that, unlike his, it is consistent not only with phenomena of heat radiation but with ordinary optical phenomena as well and with limitations of amplitude imposed by known sizes of atoms, and that besides correlating the phenomena of heat radiation and optics, it connects them also with those of magnetism, atomic volume relations, and chemical affinities.

In Page's theory the exact nature of the oscillator is not specified, but it is suggested that it might be a rotating electron or group of electrons. Nothing stated there would prevent one from changing that assumption to make it the magneton. The first radical difference between the two theories is that in Page's the oscillations are assumed to be stable only if their energy, without regard to that of the rotation, is very close to an integral multiple of $h\nu$. This assumption involves two serious difficulties. First, as I have pointed out in connection with Planck's theory, such a vibration involves amplitudes that are apparently too large for the atom; and second, it is not evident how such a restriction to definite energies of vibration can agree with the optical phenomena explained by the ordinary dispersion theory. Since the opportunity to correlate heat radiation with these optical phenomena, as my theory does, seems to be really the chief object to be gained by combining the classical electrodynamics and the quantum hypothesis, this point appears important in deciding between the two theories.

With regard to the internal mechanism of transfer, Page assumes that it acts in such a way as to neutralize any difference between the rates of radiation and absorption by the oscillations, keeping them constant except in collisions and placing the gain or loss in the rotation. Then he calculates the entropy of the system from Planck's postulate $S = k \log W$,

determining the microscopic state by the energy of each oscillation, without regard to what is stored in the corresponding rotation. Thus this theory appears to gain in simplicity by eliminating the assumption $(1 - \eta)/\eta = pI$ made in Planck's theory and mine.

There is, however, an important difference between the way this equation $S = k \log W$ is used in Page's theory and in the others. For as Planck has pointed out,[1] this postulate cannot be true unless all possible microscopic states are equally probable, since otherwise the most probable macroscopic state will not be the one with the largest value of W, and the entropy thus defined will not be a maximum. In the case of Planck's theory and mine, the postulate is justifiable by a proof that this equality of probabilities is a direct result of the previous assumptions, or by the proof given by Planck that the same distribution results from this postulate as from the previous assumptions. In Page's theory, on the other hand, the equality of probabilities is really a new assumption implied in the postulate $S = k \log W$.

The question now is, Is this assumption more or less reasonable than Planck's $(1 - \eta)/\eta = pI$. If the oscillators were independent conservative systems, exchanging energy only by quanta, the implied assumption would amount only to the condition that each one could gain or lose a quantum with a readiness independent of the number already in its possession, so long as it did not have to go below zero. But since the oscillators are not conservative, there will always be some, radiating faster than they absorb, that must stop oscillating for lack of a supply of stored energy. Consequently others, absorbing faster than they radiate, must be started into larger vibrations. This introduces the necessity for some sort of a law of starting oscillations from the stored energy. This law might take a form somewhat like the repetition of the condition $(1 - \eta)/\eta = pI$ on passing each multiple of the quantum in the storage system.

Now this condition, as applied by Planck, seems extremely plausible. For one might readily suppose the probability of radiation, η, if small, to be proportional to the time required for the stored energy to get through the neighborhood of the value $nh\nu$, that is, inversely to I. On the other hand if the probability, $1 - \eta$, of getting through, is small, one might assume it proportional to the speed of acquiring energy, and therefore to I. The simplest way to combine these assumptions is Planck's $(1 - \eta)/\eta = pI$. One might suppose it would be possible to apply this equation, in some form, to the assumption implied in Page's theory, but

[1] Heat Radiation, translation by Masius, 1914, p. 125.

an attempt which I have made to do so led to a value of η dependent on n, and I do not at present see any way to avoid this, although there may be one.

The net result, then, is that Page's theory shows an apparent inconsistency with data on atomic magnitudes, does not cover such a large range of phenomena as mine, and involves at least as many radical assumptions, if not more.

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THE PHOTO-ELECTRIC CURRENT AS A FUNCTION OF THE ANGLE OF EMISSION AND THE THICKNESS OF THE EMITTING FILM.

BY WILLARD GARDNER.

RUTHERFORD[1] has observed that for a thin deposit of radioactive matter the distribution of the active discharge about the normal to the surface indicates a uniformity of emission at all angles from any given very small element of volume. Schmidt,[2] on the other hand, has taken into consideration the effect of absorption in the case of a thick film of active substance and finds a resultant effective intensity roughly proportional to the cosine of the angle of emergence of the active particles.

In the case of photo-electric emission, it would seem from the work of Stuhlman[3] and Robinson,[4] with thin metallic films, that the activity of the exciting light is not confined to the immediate surface of the metal, but that a finite thickness is effective in the production of photo-electrons. Ladenberg[5] concludes that the effective light penetrates to a depth of eight wave-lengths. In his Physical Optics, Wood has given figures indicating that throughout the ultra-violet region a large percentage of the incident light is absorbed by metals.

On account of their high velocities, the β and cathode particles are able to penetrate a considerable distance through absorbing media. For photo-electrons, Lenard[6] has recorded velocities ranging from 10^7 to 10^8 cm. per second from carbon, platinum, and aluminum. The penetrating power is thus much less than that of β and cathode particles, but no doubt the photo-electrons emerge from a finite thickness of metal and present a similar case to that of the emission from thick films of radioactive matter. The velocity distribution curves of Richardson and Compton,[7] considered in connection with their theory, would also indicate a falling off in velocity due to a penetration of varying thickness of material.

[1] Rad. Sub. and Their Rad., p. 312.
[2] Phys. Zeit., 9, p. 537, 1908.
[3] PHYS. REV., 32, p. 621, 1911; Phil. Mag., 20, p. 331; 22, p. 854.
[4] Phil. Mag., 23, p. 542, 1912; 25, p. 115, 1913.
[5] Ann. d. Physik, 12, 3 p. 573.
[6] Allen, Photo-Electricity, p. 38.
[7] Phil. Mag., Vol. 24, pp. 575–594 (1912).

Lenard[1] has observed a variation of intensity at different angles of emission. Hughes[2] concludes that the emission is the same at all angles, though his conclusion is an inference from an indirect observation. Robinson[3] has attempted to ascertain the approximate limiting angle (maximum angle from the normal) at which photo-electrons are emitted, in order to test the formula given by Riecke. Riecke assumes that within a given cone about the normal the intensity of emission is independent of the angle and that outside of this cone there is a sudden drop in the intensity. Robinson's observations indicate only an approximate verification of this assumption. He finds from his method that the limiting angle for zinc and silver is in the neighborhood of 30°, whereas, in order to satisfy the equation of Riecke, a much larger angle is demanded. It is quite evident, however, from the data recorded, that there is a falling off in intensity with increasing angle, although the investigation gives only approximate values and for a small angular range about the normal.

The present investigation was undertaken with the view of making a more accurate determination of the angular distribution of the photo-electric current from a small flat metal plate, with varying thickness of effective substance.

The accompanying sketch (Fig. 1) gives an outline of the experimental arrangement. A small silver plate A, connected with the negative ter-

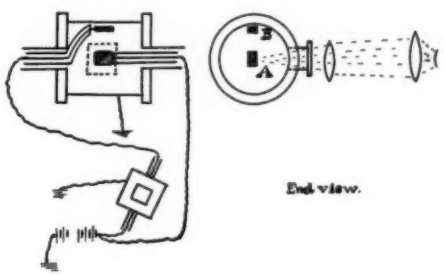

Fig. 1.

minal of a battery of cells of about 320 volts, was introduced through a ground glass joint at one end of a small metallic cylinder, through the other end of which was similarly introduced another small insulated electrode ("feeler") B, which was connected with the leaf of a Wilson tilted-leaf electroscope, and so adjusted as to rotate about A at a distance of approximately 4 cm. A beam of light from a Fuess mercury arc was

[1] Ann. d. Phys., 2, p. 359 (1900); 8, p. 159 (1902).
[2] Ann. d. Phys., 12, pp. 453, 777 (1903).
[3] Ann. d. Phys., 31, p. 769 (1910).

admitted through a quartz window at the side of the cylinder and allowed to fall normally upon the plate A. Before each observation electrode B was connected to earth and then insulated and the intensity of the electron stream at any given angle was determined by measuring the rate of fall of the electroscope leaf. The cylinder was connected to earth, and, as electrode B was made to rotate close to the walls of the cylinder, the initial field within was practically radial in the section containing the electrodes. The inside of the cylinder was coated with soot to avoid the photo-electric effect from reflected light.

For the work with thin films, a quartz plate was substituted for the metal and the films were deposited by the method of cathode sputtering with an apparatus designed by Prof. Lewis for silvering his interferometer plates. The films used were of a beautiful uniformity in appearance.

In Fig. 2 (a) is shown a curve representing the mean of a large number of observations for a thick plate at pressures ranging from 0.1

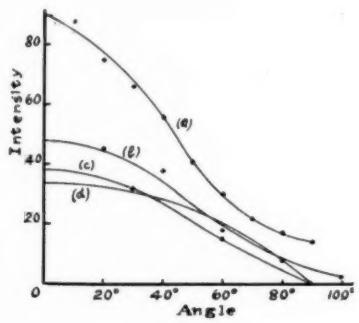

Fig. 2.

to 0.01 mm. of mercury. The precision with which the points fall upon the line is sufficient indication of the degree of accuracy or the mean values.

Curve (b), Fig. 2, represents the mean of several observations taken at one time, for pressures below 0.008 mm. Curve (c) represents the mean value of a series of five observations taken at an extremely high vacuum obtained by the use of liquid air. As judged by the length of spark gap in air in parallel with a gap through the vessel, the pressure was considerably below 0.001 mm. In fact, it was almost impossible to get a discharge to pass through the vacuum. The photo-electric current for these low values of the pressure was extremely small, but the results were perfectly definite. The probable error of the mean for each point was calculated and for the three points given was less than 8 per cent.,

the probable error of any single observation being from 10 to 14 per cent. Owing to the difficulty of maintaining extremely high vacuum over long intervals of time and to the fact that the currents were so extremely small, three points only were determined. The general form of the curve is shown by (*a*), and the particular purpose of the observations at low pressures was to verify the supposition that the finite ordinate in (*a*) at 90° was due to scattering. The relative values of the total current for varying pressures were found to harmonize very well with values reported by Varley.[1] For convenience in plotting, accurate relative values of the ordinates are not given. The intensities of the current for curve (*a*) were much greater in comparison with (*b*) and (*c*) than the plot would indicate.

It is true of course that the greater part of the current measured at the higher pressures was due to ions produced by the emitted electrons. Inasmuch, however, as the field was practically radial, it is to be expected that the direction of the resulting total current would correspond in general with the initial direction of the emitted electrons. Under the influence of a uniform radial field any modification of the character of the angular distribution that might occur would be toward uniformity at all angles, and the finite ordinate at 90° in the curves for higher pressures might be thus accounted for. It may also be observed that the slight departure from a uniform radial field, due to the manner of construction of the apparatus, would also account for a slight distortion, the field being somewhat greater in the neighborhood of 90°.

In Fig. 3 (*a*), is shown the result of a large number of observations taken with thin films of varying thickness, ranging from 20 to 200 $\mu\mu$, taken under conditions of pressure ranging from 0.1 to 0.01 mm. Accurate determination of the film thickness was not attempted. A number of films were weighed and the approximate thickness also determined by means of the spectrophotometer, a rough calibration of the sputtering apparatus being thus accomplished.

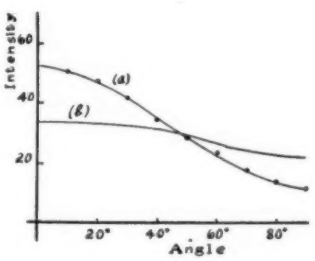

Fig. 3.

In order to explain upon theoretical grounds the results recorded above, it will be assumed that the electrons are emitted equally in all directions from the molecule or infinitesimal element. While the work of Stuhlman and Robinson[2] would seem to indicate that such may not be the case,

[1] Allen, Photo-Electricity, p. 58.

[2] Loc. cit.

yet, in the light of work reported by Hallwachs[1] showing corresponding asymmetry in the actual amount of light absorbed, it would seem that the matter is still an open question, and for the lack of a more definite knowledge regarding this point, it is perhaps not inappropriate to make this assumption as a working hypothesis.

It will also be assumed that the law of absorption of the electrons is logarithmic, as well as the law of absorption of the light.

It should be observed that the method of measurement used in this work gives the total current converging at the " feeler " from the entire volume of emitting substance, and the angular dimension of the converging beam varies with the position of the " feeler." The results reported do not, therefore, purport to be a measurement of what may be called a " parallel " beam in each case, but a measurement of the number of electrons converging at the " feeler " from the entire volume of emitting substance. Therefore, in the formation of the equation, it will be necessary to integrate over the entire volume of metal and not over a given solid angle at the " feeler."

It has been shown by various investigators, viz., Elster and Geitel, Lenard, Hermann and Richtmyer, and others, that for any given wavelength the number of electrons emitted is directly proportional to the intensity of the exciting light, although slight departure from this law has been noted by Ives.[2]

Let us, therefore, assume an equal emission in all directions from the molecule, a logarithmic absorption of the electrons with absorption coefficient k_1, a logarithmic absorption of the light with absorption coefficient k_2, and an actual number of electrons emitted from the volume element proportional to the volume and to the intensity of light within the volume, with proportionality factor C, *i. e.*, the number of electrons emitted per unit intensity of light per unit volume.

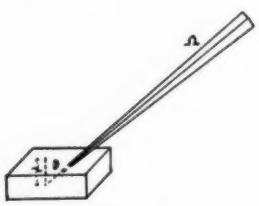

Fig. 4.

Then, if I_0 is the intensity of the light at the surface, the intensity I at a point at depth x in the metal is given by the equation,

$$I = I_0 e^{-k_2 x}.$$

The number of electrons n emitted per unit volume will be

$$n = CI_0 e^{-k_2 x},$$

and for the radiation from a volume element $dxdydz$ through a given solid angle Ω (see Fig. 4), we have,

[1] Allen, Photo-Electricity, p. 47.

[2] PHYS. REV., Vol. III., p. 68, 396 (1914).

$$dN = CI_0 \frac{\Omega}{4\pi} e^{-k_2 x} dxdydz,$$

Under the experimental conditions the angle Ω subtended by the "feeler" at the point of emission was practically constant over the entire volume and the only variable involved in the integration is the depth x measured normally into the metal.

The length of path traversed in the metal before emergence for a beam Ω traveling in a direction defined by the angle θ measured from the normal (azimuth angle = 0) is equal to x sec θ, and the number dN_θ emerging through Ω at an angle θ will be given as follows:

$$dN_\theta = CI_0 e^{-(k_1 \sec\theta + k_2)x} \frac{\Omega}{4\pi} dxdydz,$$

and the total for the entire plate becomes,

$$N_\theta = \int_0^{x2} CI_0 e^{-(k_1 \sec\theta + k_2)x} \frac{\Omega}{4\pi} dxdydz.$$

Writing,

$$CI_0 \int \frac{\Omega}{4\pi} dydz = A,$$

the integral becomes,

$$(A) \qquad N_\theta = \frac{A \cos\theta}{k_1 + k_2 \cos\theta} (1 - e^{-(k_1 \sec\theta + k_2)x_1}).$$

For a thick plate the exponential term vanishes, giving

$$N_\theta = \frac{A \cos\theta}{k_1 + k \cos\theta}$$

from the illuminated side of the plate. For thin films a similar development gives for the emission at the rear of the film,

$$(B) \qquad N_\theta = \frac{A \cos\theta}{k_2 \cos\theta - k_1} (e^{-k_1 x_1 \sec\theta} - e^{-k_2 x_1}).$$

In Fig. 2 (*d*) is represented the distribution based upon the equation given for a thick plate with an arbitrary selection of constants ($k_1 = 2$, $k_2 = 1$), and there is seen to be a fairly close agreement in the shape of the curve with the experimental curve for high vacuum, except that the point of inflection, though less prominent than in the curves for higher

pressures, still remains in the latter. Allowing for an experimental error of the magnitude noted above, this change of curvature may be made to disappear. Inasmuch, however, as this feature is present in all the curves, it may be significant in this case.

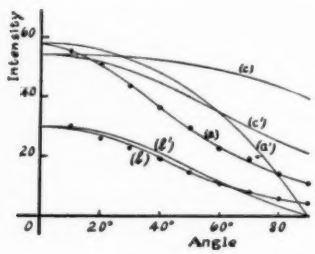

Fig. 5.

In Fig. 5 (a) is shown the experimental curve obtained with a thin film, the distribution measured on the illuminated side; (b) represents the distribution measured on the other side. Curve (c) shows the ratio of the ordinates of (b) and (a). Curves (a'), (b') and (c') represent the corresponding curves plotted from equations (A), (B) and the ratio $(B)/(A)$, where $k_1 = 2$, $k_2 = 1$, and $x_1 = 2$. As stated, the unit of intensity for each curve is arbitrary and the value of the ratio of the ordinates represents only approximately the actual experimental results.

It may be readily shown by forming the derivatives that the functions (A), (B) and $(B)/(A)$ are all diminishing functions of θ. The derivatives are somewhat cumbersome and are therefore omitted, but the curves (a'), (b') and (c') of Fig. 5 will serve to illustrate this point, and are seen to harmonize in general with the corresponding experimental curves.

If we grant that there is a greater tendency for emission from the molecule or volume element in the direction of the light, such a tendency would no doubt result in a relatively larger effect normal to the surface for the " emergent " electrons, but as observed, this is also to be expected from the equations given.

In this connection it may be noted that for the thinnest film used in this work a lack of symmetry in " incident " and " emergent " effect was not observed. For this case the total current was found to be about one per cent. higher for the " incident " effect. However, the total current was measured only in one case and the observation was merely incidental. In view of the fact that so much experimental evidence has been obtained showing that the velocity of emission of the photo-electrons depends entirely upon the frequency of the exciting light,[1] it is very difficult to conceive of any intrinsic influence other than the pressure of the light that would give higher velocities to the " emergent " electrons. It cannot, for example, be conceded that no electrons are emitted from the surface molecules in the direction opposed to the direction of the light, and if the initial velocity is dependent upon the frequency of the light alone, the ultimate potentials for the " incident " and the " emergent "

[1] Millikan, PHYS. REV., March, 1916, p. 355.

effect can differ only by the amount due to the pressure of the light, combined with causes due to the difference in the optical conditions for the "incident" and "emergent" effect.

As may be seen from the manner of the derivation of equations (A) and (B), the rate of change of the intensity of the photo-electric current with the thickness of the film involves the angle θ. Forming the derivative with respect to x_1 and then again taking the derivative with respect to θ, we have,

$$\frac{dN_\theta}{dx_1} = Ae^{-(k_1 \sec\theta + k_2)x_1},$$

and

$$\frac{d\,(dN_\theta)}{d\theta(dx_1)} = -Ak_1x_1 \sec\theta \tan\theta e^{-(k_1 \sec\theta + k_2)x_1}.$$

The change of N_θ with film thickness is seen to be a positive function of x_1 and θ, but the derivative of the latter function with respect to θ is a negative function of x_1 and θ, indicating that the intensity of the photo-electric current increases faster with the film thickness at small values of θ. We should expect, therefore, that the ratio of the ordinates for the thick plate to those for the thin film should decrease with θ. In Fig. 3 (b) is plotted the ratio of the ordinates of (a) Fig. 2 (thick plate) to those of (a) Fig. 3 (thin film), and the results are seen to be in perfect harmony with the equations.

In plotting the theoretical curves the constants and the film thickness are arbitrarily assumed and general qualitative agreement is all that can be expected. The results cannot be taken as conclusive evidence against the claims of Stuhlman and Robinson, but the qualitative agreement in the general shape of the calculated and experimental curves would seem to indicate that the fundamental assumptions made are at least not far from the truth. On the other hand, if there is an actual lack of symmetry in the emission from the molecule, we must await further experimental work before an accurate theory can be proposed.

It is interesting to note the form assumed by the function if we consider the intensity of light constant throughout the metal and take account only of a logarithmic absorption of the consequent radiation (corpuscular or otherwise), a case realized for incandescent substance emitting temperature radiation. In this case the function may be represented in the form of a converging series,

$$N_\theta = Ax_1\left(1 - \frac{1}{2}k_1x_1 \sec\theta + \frac{1}{3}\frac{k_1^2x_1^2 \sec^2\theta}{|2} - \frac{1}{4}\frac{k_1^2x_1^2 \sec^2\theta}{|3} + - \cdots\right),$$

which is seen to be independent of θ for small values of x_1. Again, if k_1 is also equal to zero, the resultant radiation converging at a point is independent of the angle from the surface normal.

In conclusion, I wish to thank Prof. E. P. Lewis, under whose direction the work has been carried on, for his many valuable suggestions.

PHYSICAL LABORATORY,
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May, 1916.

THE HALL EFFECT AND ALLIED PHENOMENA IN RARE METALS AND ALLOYS.

BY ALPHEUS W. SMITH.

IT is important for the development of the theory covering the Hall effect and allied phenomena to extend the investigation of these phenomena to as many metals and alloys as possible. Although the data in this region are already very extensive, several of the less well known and less easily obtained metals have not been studied. It is the purpose of this investigation to make a study of these phenomena in some of the rarer metals which can be obtained in a form suitable for such an investigation. The metals studied are molybdenum, tungsten, indium, cerium, tantalum, thallium, lithium, and iron-cobalt alloy. The molybdenum and tungsten were kindly sent to me by the General Electric Company. The indium, cerium, thallium and lithium were obtained from Merck & Company. The tantalum came from Kahlbaum. The iron-cobalt alloy was kindly given me by Mr. Williams, of the University of Illinois. The iron and cobalt entered into the alloy in the proportions indicated by the formula Fe_2Co. Williams[1] has shown that this alloy has a higher value of the intensity of magnetization than either iron or cobalt.

The metals were in the form of plates about 1.5 cm. wide and 4.0 cm. long. Narrow arms were cut so as to project from the sides of the plates. These arms served in the usual way for Hall electrodes and made corrections for the error due to the Ettingshausen effect unnecessary. Copper wires were soldered to the plates to serve as leads for the primary current and for leads from the Hall electrodes to the galvanometer. Where necessary in order to connect the leads to the wires by soldering, the ends of the plate and the ends of the arms were copper plated over the entire length and the leads then soldered to these copper-plated regions.

The observations on the electromotive force set up by the magnetic action were made by noting the deflection of the galvanometer when the field was first in one direction and then in the other direction. A series of such observations was taken for each plate. From this mean value together with the sensibility of the galvanometer the desired difference of potential was in each case determined. The temperature gradient in the plate in the case of the Nernst effect was determined by two copper-advance thermal couples soldered to the plate at two points on its longi-

[1] PHYS. REV. (2), 6, p. 404 (1915).

tudinal axis at a distance of about 2.0 cm. from each other. The observations for the Hall effect were made at room temperature which was about 24° C. The magnetic field had a magnitude of about 17,000 or 18,000 C.G.S. units. To get the temperature at which the Nernst effect was observed, the mean of the temperatures of the junctions at the ends of the plate was taken. This mean temperature did not differ by more than one or two degrees for 60° C. and this is taken as the temperature at which the Nernst effect was determined. The magnetic field had the same value in these observations as in those on the Hall effect.

The coefficient of the Hall effect was calculated from the familiar equation,

$$E = {}_eT_e \frac{Hi}{d},$$

where E is Hall electromotive force in absolute units; H the magnetic field and i the current in absolute units; d is the thickness of the plate in centimeters. The coefficient of the Hall effect has been denoted by ${}_eT_e$ following the notation suggested by Professor Hall.[1] The T calls attention to the fact that the observed effect is transverse. The two subscripts (e) indicate that both the longitudinal flow and the transverse flow are electrical. In the same way the coefficient of the Nernst effect has been calculated from the equation

$$E = {}_hT_e \beta H \frac{\partial t}{\partial x}.$$

Here β is the width of the plate in centimeters; H, the magnetic field in absolute units; $\partial t/\partial x$, the longitudinal temperature gradient in the plate in degrees Centigrade per centimeter; E is the observed electromotive force in absolute units. The coefficient has been denoted by ${}_hT_e$ instead

TABLE I.

Metal.	${}_eT_e \times 10^4$.	${}_hT_e \times 10^5$.	Thermoelectric Height.
Indium	− 0.73	+ 3.2	+
Lithium	− 17.0	+ 16.0	−
Molybdenum	+ 12.6	− 17.2	+
Tungsten	+ 11.8	−100.0	±
Cerium	+ 19.2	+ 10.0	+
Tantalum	+ 10.1	+ 9.8	−
Thallium	+ 2.4	− 3.7	+
Fe_2Co	+ 13.3	+250.0	−
Iron	+110.0	− 81.0	+
Cobalt	+ 24.0	+200.0	−

[1] Proc. Am. Acad. of Arts and Sci., 46, p. 625 (1911).

of by Q which usually occurs in the literature on this subject. The subscript h before the T indicates that the longitudinal flow in the plate is thermal and the subscript e, that the observed transverse effect is electrical.

In the second column of Table I. the coefficients of the Hall effect and in the third column the coefficients of the Nernst effect have been given. The fourth column gives the direction of the thermoelectromotive forces in these metals against lead when the cold junction is at 0° C. and the hot junction at 100° C. Coblentz[1] has shown that tungsten against copper has a temperature of inversion in the neighborhood of 40° C. Its thermoelectric height against copper would be negative for the temperature at which the Hall effect was observed and positive for the temperature at which the Nernst effect was observed. While there is some intimate relation between the direction of the Hall effect, the Nernst effect and the thermoelectric height of the metal it is evident from Table I. that this relation is not simple. With the customary conventions with respect to signs the Hall effect and the thermoelectric heights have the same sign but the Nernst effect and the thermoelectric heights have opposite signs. There are, however, several exceptions to these two rules. These exceptions may in part be explained by the fact that the thermoelectric heights are arbitrarily taken against lead. Koenigsberger[1] and Gottstein have undertaken to make this comparison by taking the difference between the constant of the Hall effect for the metal in question and the same constant for gold, and comparing this difference with the thermoelectric height of the metal against gold. This method removes some, but not all, of the discrepancies.

The values of the Hall constant and the Nernst constant for the alloy Fe_2Co are of especial interest in connection with the theory that the reversal of the Hall effect is due to molecular fields in the neighborhood of the atoms. These molecular fields which are in the opposite direction to the external magnetic field would cause the electrons to be deflected in the opposite direction to that observed in bismuth which is the direction predicted by the electron theory. Now this alloy has a greater intensity of magnetization than either iron or cobalt so that if these internal molecular fields are effective in determining the magnitude and direction of the effect the constant of the Hall effect for this alloy would be larger than it is in either iron or cobalt and the direction the same as in these metals. In order to make a comparison of these constants for this alloy with the corresponding constants for the components, iron and cobalt, the value of these constants for iron and cobalt have been

[1] Bull. Bureau of Standards 6. p. 107 (1909–10).
[2] Phys. Ztschr. 14. p. 232 (1913).

included in Table I. An inspection of this table shows that the direction of the Hall effect in this alloy is the same as its direction in iron and cobalt, but its magnitude is only about one half that in cobalt and about one eighth of that in iron. The Nernst effect has the direction which it has in cobalt, and about the same magnitude. Its direction, however, is opposite to that in iron and its magnitude more than three times that in iron. The thermoelectromotive force against lead has the same direction as the thermoelectromotive force of cobalt which is opposite to iron. It is seen from this that the addition of cobalt to the iron causes a reversal of the thermoelectric height, a reversal of the Nernst effect and a decrease of the Hall effect. The alloy behaves like cobalt, although there is nearly twice as much iron as cobalt in it. The fact that one of these effects may be reversed without the reversal of the other indicates that a complete explanation of the reversal of these effects is not to be found in the molecular fields about the atoms.

THE ETTINGSHAUSEN EFFECT IN ALLOYS.

When a plate carrying a current of electricity is placed in a magnetic field in such a way that the plane of the plate is perpendicular to the magnetic field, a difference of temperature is set up between the edges A and B of the plate, Fig. 1. The magnitude of this difference of temperature is determined by the current in the plate, the thickness of the plate and the magnetic field. Let $\Delta\theta$ be the difference of temperature between A and B due to the magnetic action; d, the thickness of the plate in cm. and i, the current in the plate in absolute units, then

$$\Delta\theta = {}_eT_h \frac{Hi}{d},$$

where ${}_eT_h$ stands for the coefficient of the Ettinghausen effect. The T indicates that the effect is transverse, the prefix e that the longitudinal flow is electrical and the subscript h that the transverse flow is thermal. The coefficient ${}_eT_h$ is said to be positive if the transverse current of heat is from A to B in Fig. 1, when the direction of the electrical current is from right to left, and the magnetic field is directed into the plane of the paper. This means that the edge of the plate which has its potential decreased in the Hall effect also becomes warmer.

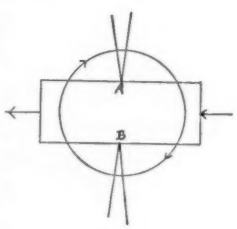

Fig. 1.

The lead was obtained from Kahlbaum, the bismuth, tin, cadmium and antimony from Baker Chemical Company. The metals were mixed

in known proportions by weight in a glass tube; the air was exhausted and the tube sealed. The metals were then fused together to form the alloy. Care was taken to insure satisfactory mixing. After the preparation of the alloys, they were again melted and poured quickly into a mould made of lavite so that a rectangular plate about 4 cm. long, 2 cm. wide, and 0.17 cm. thick was formed. This mould was at room temperature when the molten alloy was poured into it. Hence the alloys were cooled quickly from the liquid to the solid condition.

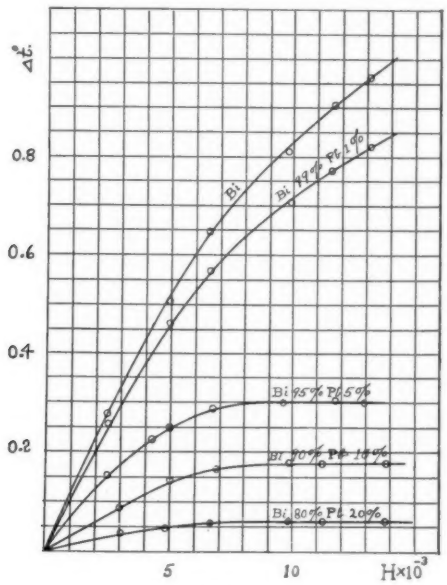

Fig. 2.

In order to protect the plate from thermal disturbances it was enclosed in a thin rectangular box made of hard rubber. This box could be inserted between the pole pieces of the electromagnet. Between the walls of the box and the plate was a layer of magnesia oxide about 0.2 cm. thick. The plate was therefore protected from the pole pieces by a sheet of hard rubber 0.3 cm. thick and a layer of magnesia oxide 0.2 cm. thick. This afforded satisfactory insulation from the external thermal disturbances.

The differences of temperature between the points A and B was determined by soldering a thermal junction of advance against copper at A and another at B. Each of these couples was connected differentially to one pair of the coils of a sensitive galvanometer, so that an increase of temperature of one junction and a decrease of temperature of the other junction tended to cause the galvanometer needle to deflect in the same

direction. Observations were made for both directions of the magnetic field. The sensibility of the galvanometer was determined for each series of observations. From these observations together with the thermoelectric height of the thermal couples the difference of temperature between the points A and B was calculated.

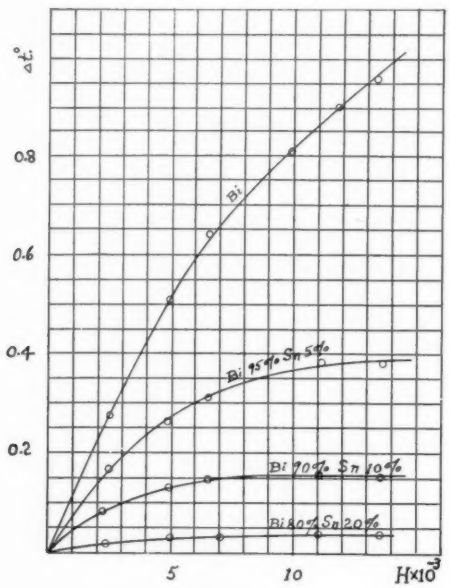

Fig. 3.

The observations were all made at room temperature which was about 23° C. On each of these series of alloys except the bismuth-antimony series observations were made at different field strengths, the lowest being about 3,000 gauss and the highest about 14,000 gauss. The results of these observations have been plotted in Figs. 2, 3, and 4. Fig. 2 shows the relation between the change of temperature and the magnetic field for the bismuth-tin alloy; Fig. 3 for the bismuth-lead alloys and Fig. 4 for the bismuth-cadmium alloys. In each of these figures the magnetic field has been plotted for abscissæ, and the change in temperature due to the magnetic action for ordinates. These temperature differences have been calculated for the case of a plate 1 cm. thick with a current of one electromagnetic unit in it.

It is seen from these curves that the change in temperature observed in bismuth is not a linear function of the magnetic field. As the amount of tin or lead is increased in the alloy the departure from the linear relation becomes greater and greater. The larger the magnetic field the greater is

the departure. The addition of either tin or lead to bismuth causes a very rapid decrease in the Ettingshausen effects at fairly high field strengths. There was evidence that at lower field strengths the Ettingshausen effect for bismuth-tin alloys containing not more than one per cent. of tin may be slightly larger than it is for pure bismuth. This is in agreement with the observations of Nernst and Ettingshausen[1] who found

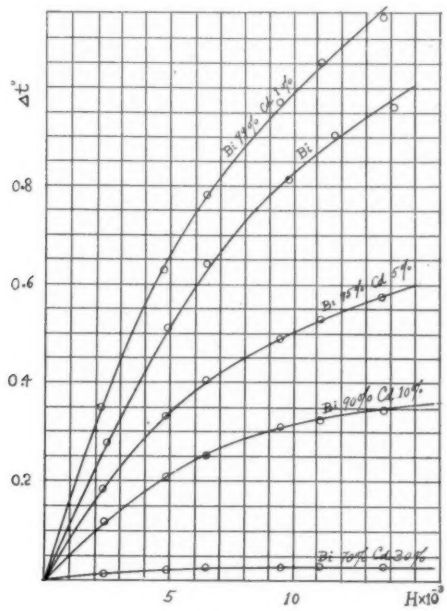

Fig. 4.

that for field strength of 4,700 C.G.S. units the addition of small quantities of tin to bismuth made the Ettingshausen effect increase then pass through a maximum for an alloy containing about 0.5 per cent. of tin, after which it decreased rapidly with further addition of tin.

The behavior of the bismuth-cadmium series of alloys is somewhat different from the behavior of the bismuth-tin and bismuth-lead series. The first addition of cadmium to the bismuth causes an increase in the Ettingshausen effect so that the curve for the alloy containing one per cent. of cadmium lies above that for pure bismuth for all values of the magnetic field. A further addition of cadmium causes a rapid decrease in this effect, so that an alloy containing 5 per cent. of cadmium has an Ettingshausen effect which is only 70 per cent. of that in pure bismuth. With the further addition of cadmium to the bismuth the effect sinks to a small fraction of its value in pure bismuth. Here as in the bismuth-tin

[1] Ann. d. Phys., 33, p. 474 (1888).

and bismuth-lead alloys the effect increases less rapidly than the magnetic field. The greater the amount of cadmium the less rapidly does the effect increase with the magnetic field.

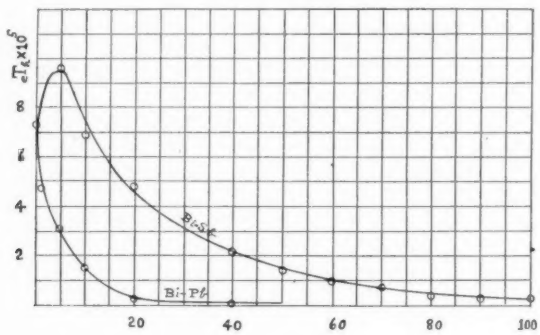

Fig. 5.

The bismuth-antimony series was studied for only a field strength of about 11,000 gauss. By reference to Fig. 5, it is seen that here as in the bismuth-cadmium the first addition of antimony causes an increase of the Ettingshausen effect. The effect passes through its maximum value when the alloy contains 5 or 6 per cent. of antimony. With further addition of antimony it sinks rapidly approaching the value in antimony.

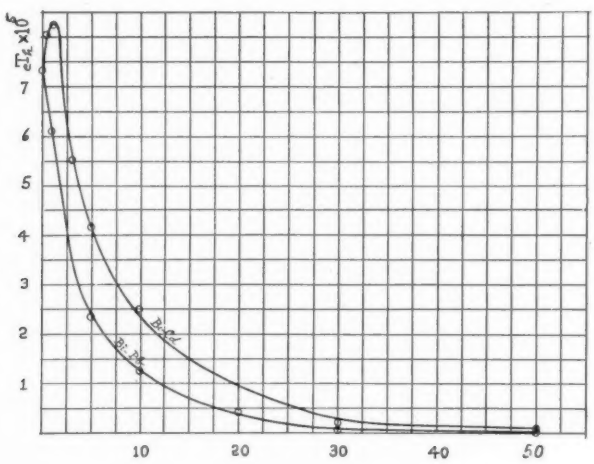

Fig. 6.

In order to bring out more clearly the relation between the concentration of the metals in the alloys and the magnitude of the Ettingshausen effect the value $_eT_h$ for each series of alloys has been plotted against the concentration by weight of the substance added to the bismuth. Fig. 6

shows the results for the bismuth-tin and for the bismuth-cadmium series and Fig. 5 the results for the bismuth-lead and the bismuth-antimony series. From these figures it is seen that bismuth-tin and bismuth-lead behave in essentially the same way; the addition of either tin or lead causing a rapid drop in the value of the Ettingshausen effect. On the other hand the bismuth-cadmium and bismuth-antimony series behave in the same way. The first addition of either cadmium or antimony causes an increase in the effect which passes through a maximum and decreases rapidly to a small part of its value in pure bismuth. The data from which these curves were plotted have been given in Table II. and Table III.

TABLE II.

Bismuth—Tin.			Bismuth—Lead.		
Bismuth.	Tin.	$_eT_h$	Bismuth.	Lead.	$_eT_h \times 10^5$
100%	0%	7.34	100%	0%	7.34
99	1	4.78	99	1	6.11
95	5	3.12	95	5	2.35
90	10	1.48	90	10	1.27
80	20	0.301	80	20	0.455
60	40	0.075	70	30	0.091
			50	50	0.033

TABLE III.

Bismuth—Cadmium.			Bismuth—Antimony.		
Bismuth.	Cadmium.	$_eT_h \times 10^5$	Bismuth.	Antimony.	$_eT_h \times 10^5$
100%	0%	7.34	100%	0%	7.34
99.5	0.5	8.06	99	5	9.67
99	1	8.27	90	10	6.92
97	3	5.51	80	20	4.72
95	5	4.16	60	40	2.20
90	10	2.51	50	50	1.44
70	30	.223	40	60	1.00
50	50	.049	30	70	.748
			20	80	.392
			10	90	.339
			0	100	.366

SUMMARY.

1. The coefficient of the Hall effect and the coefficient of the Nernst effect have determined in indium, lithium, molybdenum, tungsten, cerium, tantalum, thallium and an alloy of iron and cobalt. Except in the case of the iron-cobalt alloy these coefficients are independent of the magnetic field and have values which are of the order of magnitude of these

coefficients in such metals as copper and silver. The Nernst effect in tungsten is much larger than it is in the other metals.

2. In indium and lithium the Hall effect has the direction predicted by the electron theory but in the other metals it has the opposite direction. The direction of the Nernst effect in indium, lithium, cerium, tantalum and the alloy of iron and cobalt is in agreement with the requirements of the electron theory. In molybdenum, tungsten and thallium the direction is opposite to that indicated by the electron theory.

3. The von Ettingshausen effect has been studied in four series of alloys and it has been found that the addition of small quantities of lead or tin to bismuth causes a very rapid decrease in the magnitude of this effect. On the other hand the first additions of either cadmium or antimony to bismuth causes an increase in this effect which soon passes through a maximum and then decreases rapidly with further increase in the concentration of either cadmium or antimony.

4. The von Ettingshausen effect in alloys of bismuth with tin, lead, or cadmium is not proportional to the magnetic field and the deviation from this proportionality is the larger the greater the concentration of the tin, lead or cadmium in the alloy.

I wish to express my thanks to the Rumford Committee for a grant which covered part of the expense of this investigation.

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THE SPECIFIC INDUCTIVE CAPACITY OF CERTAIN METALS.

BY FERNANDO SANFORD.

AN electron moving in an elliptical or circular orbit around a central positive charge must have an acceleration toward the center of its orbit proportional to the ratio of its mean orbital radius to the square of its periodic time, hence its centripetal force may be written $F' \propto (R/T^2)$, where R is its orbital radius and T its periodic time.

If such electron be a source of light, its wave-length will be proportional to its periodic time, hence we may write $F' \propto (R/\lambda^2)$.

While we cannot know with certainty the orbital radii of the electrons revolving around different atoms, we may, at least, make plausible assumptions about them, and on the basis of these assumptions we may calculate the relative magnitude of the forces acting upon them.

In the spectral series which have been recognized in a considerable number of elements the wave-lengths converge toward some shortest possible wave-length according to a definite law. In the group of alkali metals three such spectral series are known for each element. In the one known as the principal series in each metal the wave-lengths toward which the series converges is less than the wave-length of any known line in the spectrum of the respective metal. It accordingly seems a plausible assumption that no electron in any of these metals may revolve around the central charge at a distance less than the orbital radius corresponding to the convergence wave-length.

If we may assume the orbital radius corresponding to this convergence wave-length as the true radius of the atom, we may calculate the relative magnitudes of the positive charges of the atoms of this group when we know their atomic radii.

If instead of the wave-length of a given spectral line we use its vibration frequency, we may write $F' \propto Rn^2$, where n is the vibration frequency. Since F' represents the centripetal force upon an electron at distance R from the central charge, if we wish to find what this centripetal force would be at unit distance from the central charge we must multiply F' by R^2, and we have $F \propto R^3n^2$. Since $R^3 \propto V$, the atomic volume, we may write $F \propto Vn^2$.

In Ann. d. Phys., 41, 524, Heydweiller has given a number of atomic diameters calculated from the volume of a gram-ion of the element in an

infinitely dilute water solution. If we assume that these diameters are equal to the orbital diameters of the innermost possible electron in the spectral series of the respective atoms, we may calculate the attraction of each central atom for an electron at unit distance. If the central charges of the atoms are equal, we should expect these forces also to be equal.

In the table below are given Heydweiller's atomic diameters, the convergence numbers and the force $F = Vn^2$ for all the elements whose atomic diameters are known and for which spectral series are given in Kayser's Handbuch. It should be remembered that it is only in the alkali metals that the convergence numbers used are regarded as belonging to the principal series of the elements. It is accordingly only to these elements that we can apply the above argument with certainty.

Element.	Convergence Number.	Atomic Diameter.	F
Li	4,358	1.75	1,016
Na	4,154	1.93	1,240
K	3,509	2.38	1,662
Rb	3,376	2.62	2,059
Cs	3,151	2.93	2,500
Mg	3,984	2.01	1,285
Ca	4,041	2.43	1,652
Sr	3,106	2.62	1,737
Ag	3,071	2.74	1,952
Cu	3,159	2.38	1,348
Zn	4,295	2.38	2,500
Cd	4,080	2.70	3,280
Tl	4,154	3.27	6,050

It is seen that the above values of F are not equal, and that accordingly the elements apparently do not have equal central charges. There is a suggestion of regularity in the increase of F from one atom to the next in the same group, so that if these values of F be plotted against the serial atomic numbers of Rydberg a marked regularity is seen in the group of the alkali metals, but not in the whole list.

It has previously been shown by the present writer that the atomic charges calculated from the mobility of ions in electrolysis vary with the serial numbers at a constant rate, but their relative values are not the same as the relative values of the central forces here calculated. This would seem to indicate, if our conclusions are correct, that the atoms must attract their electrons with forces which are not proportional to their central charges, and accordingly that they must have different specific inductive capacities.

Since the attraction between an electron and a positive charge at unit distance is given by the equation $F = Qe/k$, where Q is the magnitude of the positive charge and k is the specific inductive capacity, and since the charge e is the same for all electrons, we may write $k \propto Q/F$. Calculated in this way from the charges of the ions in electrolysis, the relative specific inductive capacities of the above elements are given in the following table.

Element.	F	Q	k
Li	1,016	3.65	359
Na	1,240	15.7	1,270
K	1,662	39.6	2,380
Rb	2,059	91.2	4,370
Cs	2,500	142	5,670
Mg	1,285	17.4	1,340
Ca	1,652	32.5	1,960
Sr	1,737	71.5	4,120
Ag	1,952	91.6	4,660
Cu	1,348	46.7	3,460
Zn	2,500	47.6	1,900
Cd	3,280	83.8	2,550
Tl	6,050	199.4	3,500

That the above numbers, which are here assumed to represent the relative specific inductive capacities of the elements, are definitely related to other atomic constants may be shown by plotting them against Rydberg's serial numbers. This is done in the accompanying curve (Fig. 1), and it will be seen that for ten of the thirteen elements given in the above table the calculated specific inductive capacities bear a constant relation to the serial numbers, while for the other three they are just half the values suggested by the curve.

This could not have been inferred from our knowledge of the spectral series, since there is no reason to conclude that for all the elements here given except the alkal metals there could not be other series with smaller convergence radii than those here used. Thus in the alkali metals the subordinate series have convergence radii 1.37 times as great as those here used.

The question as to whether the numbers here calculated actually represent the relative specific inductive capacities of their respective atoms can only be answered inferentially from the variation of properties which in non-metallic elements vary with specific inductive capacity. A comparison with the melting points of the alkali metals and with the

refraction constants of all the metals here used show quite as close an agreement as these properties show with the measured specific inductive capacity of other elements. The relation of the specific inductive capa-

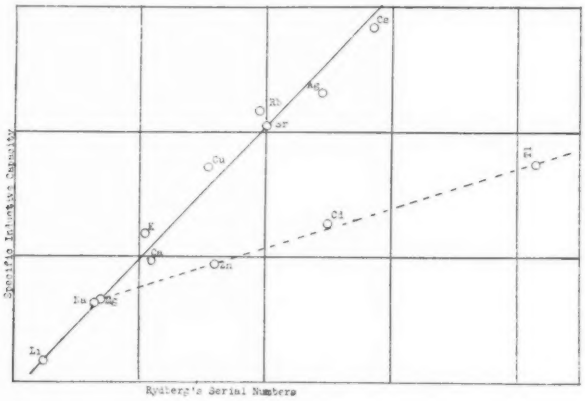

Fig. 1.

cities calculated above to Heydweiller's refraction constants for the same elements is shown in Fig. 2.

A further justification for the above values of specific inductive capacity may, perhaps, be found in their relations to Planck's law. Thus it has been found that the energy of electrons set free by the action of ultra-violet light

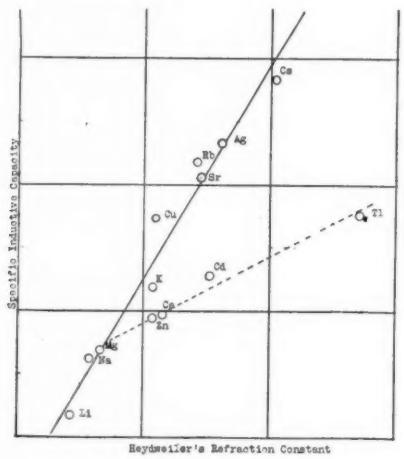

Fig. 2.

on different metals is independent of the nature of the metal and proportional to the frequency of the incident light. It seems very probable that the energy of the expelled electrons is the same as their orbital energy at the

instant of expulsion, and that their orbital frequency was before expulsion the same as the frequency of the incident light. If this is the case, the orbital energy of an electron at the instant of its expulsion would seem to have been $E = nh$, where n was its orbital frequency and h is Planck's constant.

The orbital energy of an electron is also given by the equation $E = mv^2/2$, hence $mv^2/2 = nh$ and $v^2 \propto n \propto 1/T$, where T is the periodic time. But $T \propto R/v$, where R is the orbital radius, hence $v^2 \propto v/R$, and $v \propto 1/R$. Accordingly, in order that the energy of revolving electrons may increase as their frequency, their orbital velocity must vary inversely as the orbital radius.

The centrifugal force of such an electron is $F' = v^2/R$. Substituting for v^2 its equivalent $1/R^2$, we have $F' \propto 1/R^3$. Hence, in order that the orbital energy of different electrons may vary as their frequencies, the forces between them and their central positive charges must vary as the inverse third power of their orbital radii.

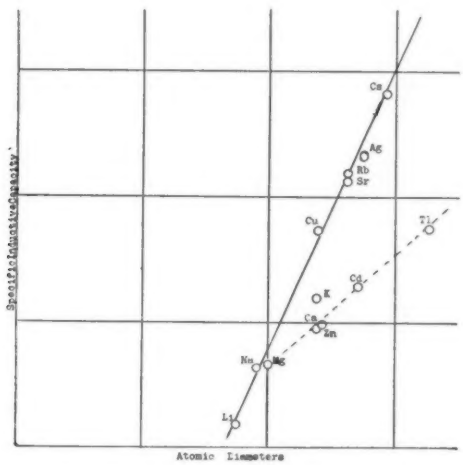

Fig. 3.

The electric attraction between an electron and its central charge must vary as the inverse second power of its orbital radius and also inversely as the specific inductive capacity of the intervening medium, *i. e.*; $F' \propto 1/kR^2$, hence for $k \propto R$, $F' \propto 1/R^3$.

That the specific inductive capacities of the different atoms here considered do vary approximately as the respective atomic radii with the same exceptions as in the case of the serial numbers is shown in the table below and in Fig. 3. This would make Planck's equation apply to different frequencies when these are caused by changing from one atom to

another of a different element. In the case of any one element the difference in the orbital radii for the different electrons which give the shorter wave-lengths is very small. Thus for the lines for which $N = 9$ and $N = 10$ in Balmer's formula while the frequency would differ by about 2 per cent., the difference in orbital radii would be less than 0.5 per cent.

Heydweiller's atomic diameters and specific inductive capacity.

Element.	Diameter.	k.
Li	1.75	35.9
Na	1.93	127
K	2.38	238
Rb	2.63	437
Cs	2.93	567
Mg	2.01	134
Ca	2.43	196
Sr	2.62	412
Ag	2.74	466
Cu	2.38	346
Zn	2.38	190
Cd	2.70	255
Tl	3.27	350

STANFORD UNIVERSITY,
March 4, 1916.

TEMPERATURE COEFFICIENT OF CONTACT POTENTIAL. A CRITICISM.

By Fernando Sanford.

IN the February, 1916, number of this journal is a paper by Professor K. T. Compton under the caption of "Temperature Coefficient of Contact Potential." In this paper Professor Compton describes an experiment on the temperature coefficient of contact electromotive force between iron and nickel which he regards as giving more accurate results than those published by Burbridge in the September, 1913, number of this journal.

Unfortunately, the apparatus used by Compton is not described with sufficient fulness in his article to enable a reader to know just what has been measured, but from the meager description given it seems to the present writer that he has not dealt at all with the contact electromotive force of the metals used.

In his description of apparatus on page 211 Compton tells us that the iron and nickel plates between which he proposes to measure the variation in contact electromotive force are "the opposing nickel and iron faces, respectively, of two hollow metal boxes." He does not tell us of what metal the boxes are constructed, but one would infer from this description that they are not nickel and iron boxes. If they are of any other metal than that of their respective face plates, it will be seen at once that as soon as the boxes are filled with water each of these plates becomes one electrode of an electrolytic cell, and its potential will be no longer the natural potential of the metal. This will be true if the boxes are made each of a single metal but have soldered joints which may come in contact with the water.

Again, after these boxes are in position their temperature is established by running water through them from some outside source. We are not told whether the water supply is insulated or in what material it is contained, but the inference from this description is that if both are not fed from the same reservoir the two reservoirs are at least connected to earth. This being the case, the two metals will be at the electrolytic potential difference of the electrodes in an iron and nickel water cell. Taking the description as it is given, the inference would seem to be that Professor Compton was measuring, not the temperature coefficient of contact potential, but that of electrolytic potential.

A word in regard to the Richardson formula upon which Professor Compton bases his faith in the inadequacy of his own results. In the formula $V_m - V_s = (1/e)(\phi_m - \phi_s) + P$, ϕ is taken as the potential energy of an electron inside and outside the given metal. That this is different in different metals indicates that there is a contact electromotive force between those metals, *i. e.*, that there is a greater force toward one metal than toward the other upon an electron in the contact surface of the two metals. The expression $(\phi_m - \phi_s)$ then becomes the measure of this contact electromotive force. If we assume, as is done in the differentiation of Richardson's formula that the quantity $(\phi_m - \phi_s)$ is a constant with reference to temperature, and that the only temperature coefficient must be that of the Peltier effect, which is in reality a very small fraction of the total contact electromotive force, we must admit that our assumption is contrary to a large amount of apparently reliable experimental data; for while the contact electromotive force is very hard to measure, its variation with temperature is very easy to observe.

STANFORD UNIVERSITY,
March 21, 1916.